Local Correction Algorithms for Low-degree Polynomials

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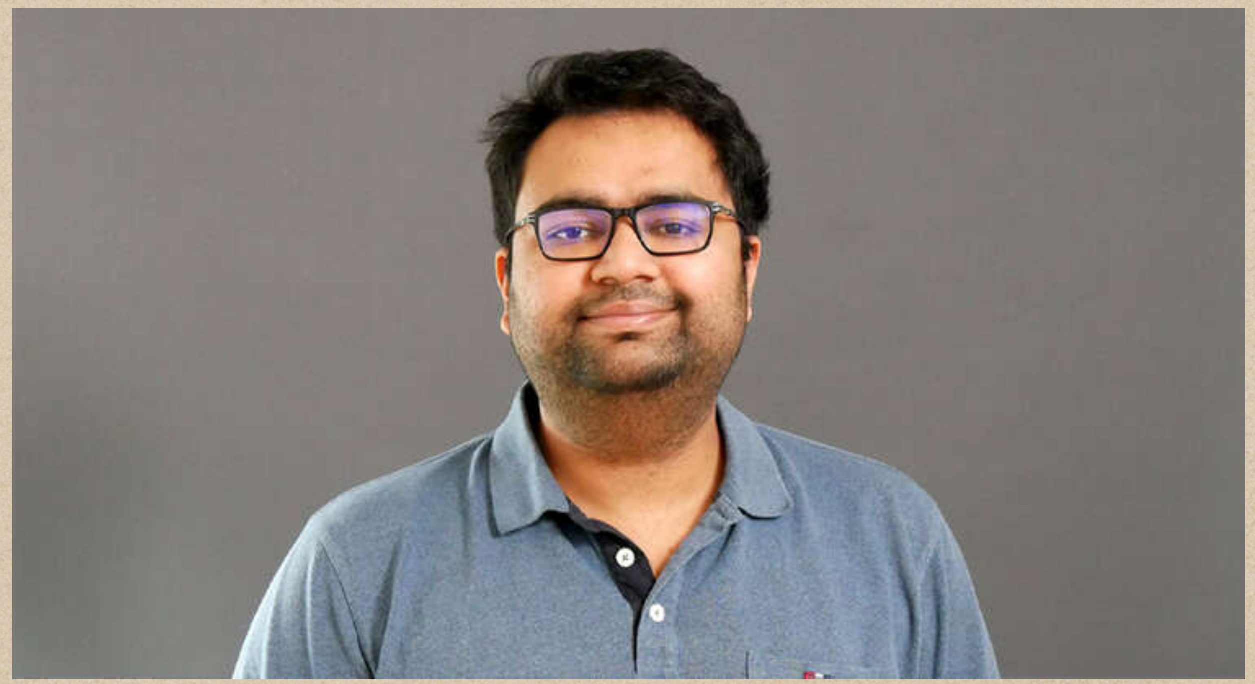
with

Prashanth Amireddy
(Harvard)



Manaswi Paraashar
(Uni. of Copenhagen)

Srikanth Srinivasan
(Uni. of Copenhagen)

Madhu Sudan
(Harvard)

What is this talk about?

Local Correction Algorithms

for

Low-Degree Polynomials

What is this talk about ?

Local Correction Algorithms

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Low-Degree Polynomials

↓

Good Codewords

What is this talk about?

Local **Correction** Algorithms

Recovering corrupted code words

for

Low - Degree Polynomials

Good Codewords

What is this talk about ?

Local

Super efficiently

Correction

Recovering corrupted code words

Algorithms

for

Low-Degree Polynomials

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What is Error Correction ?

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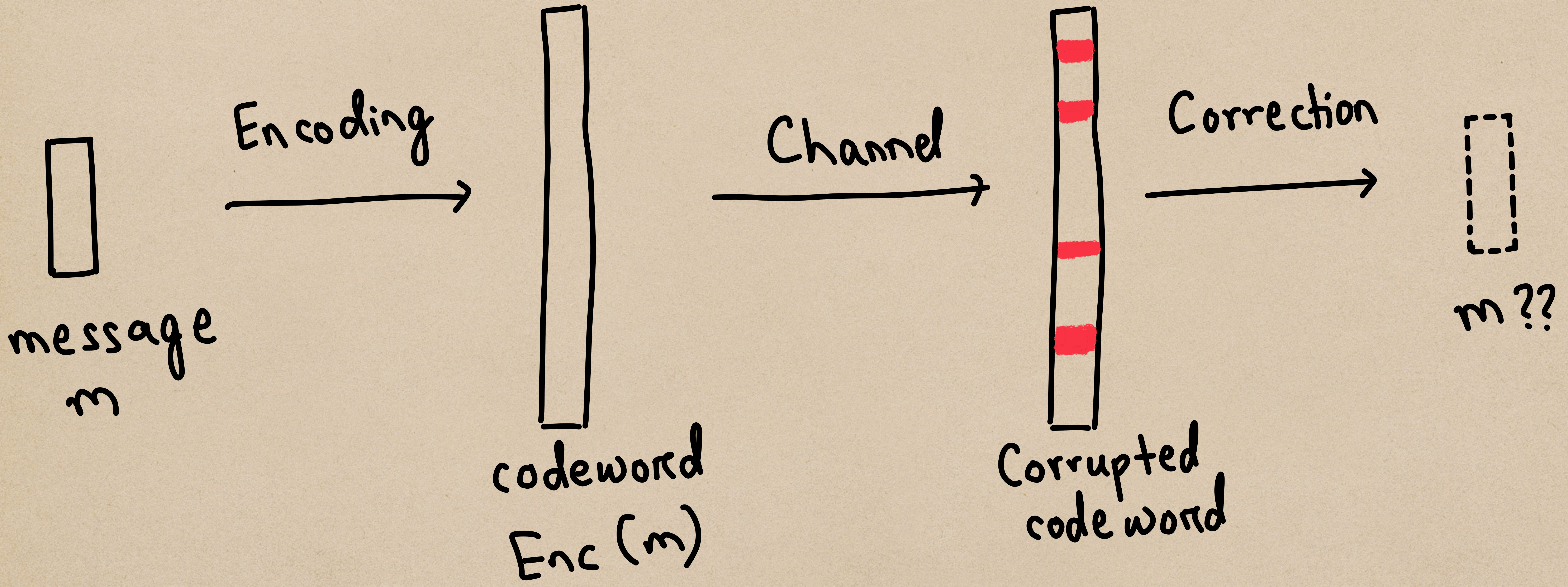

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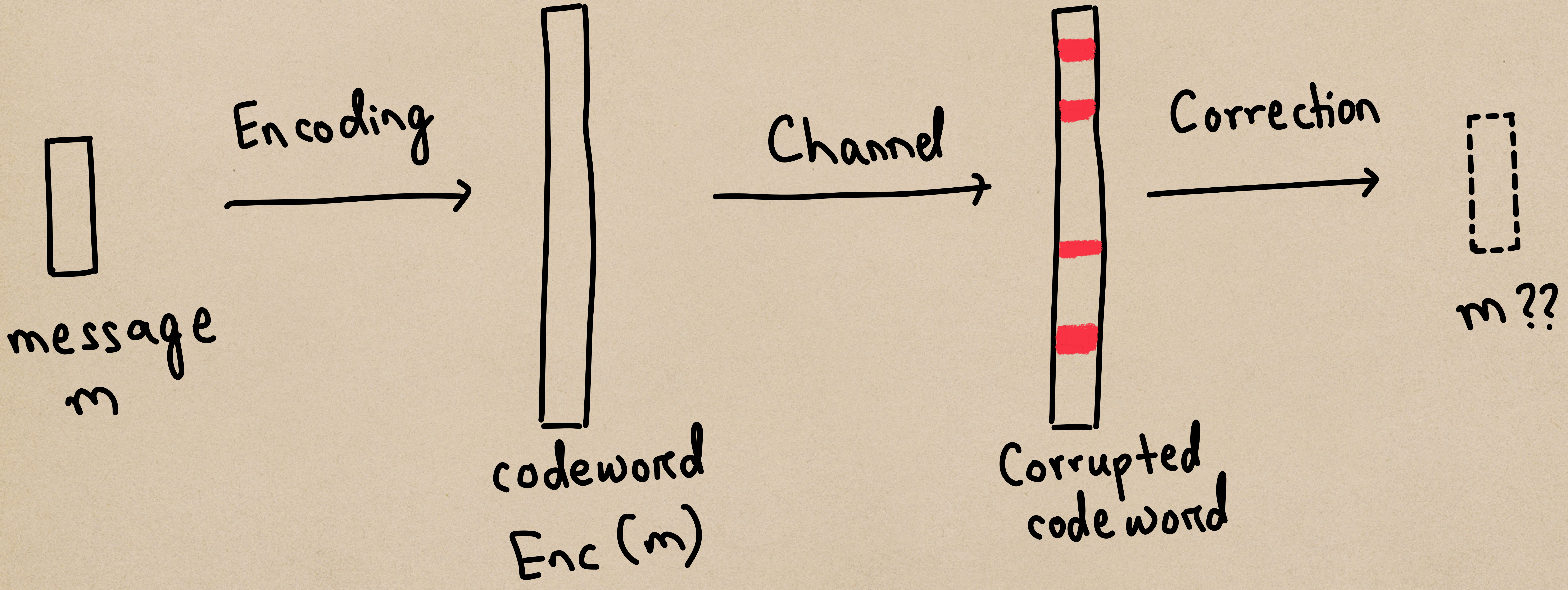
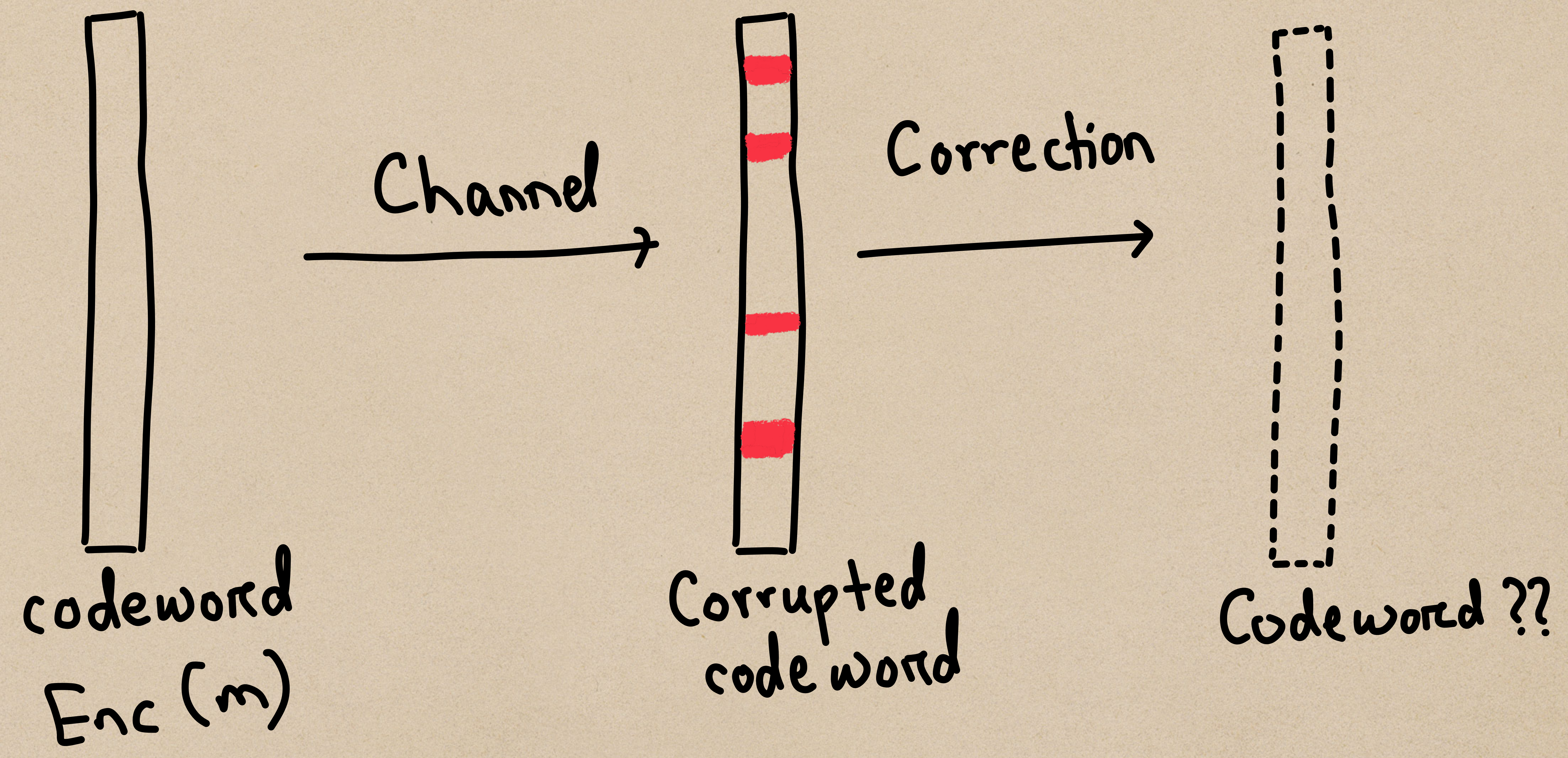


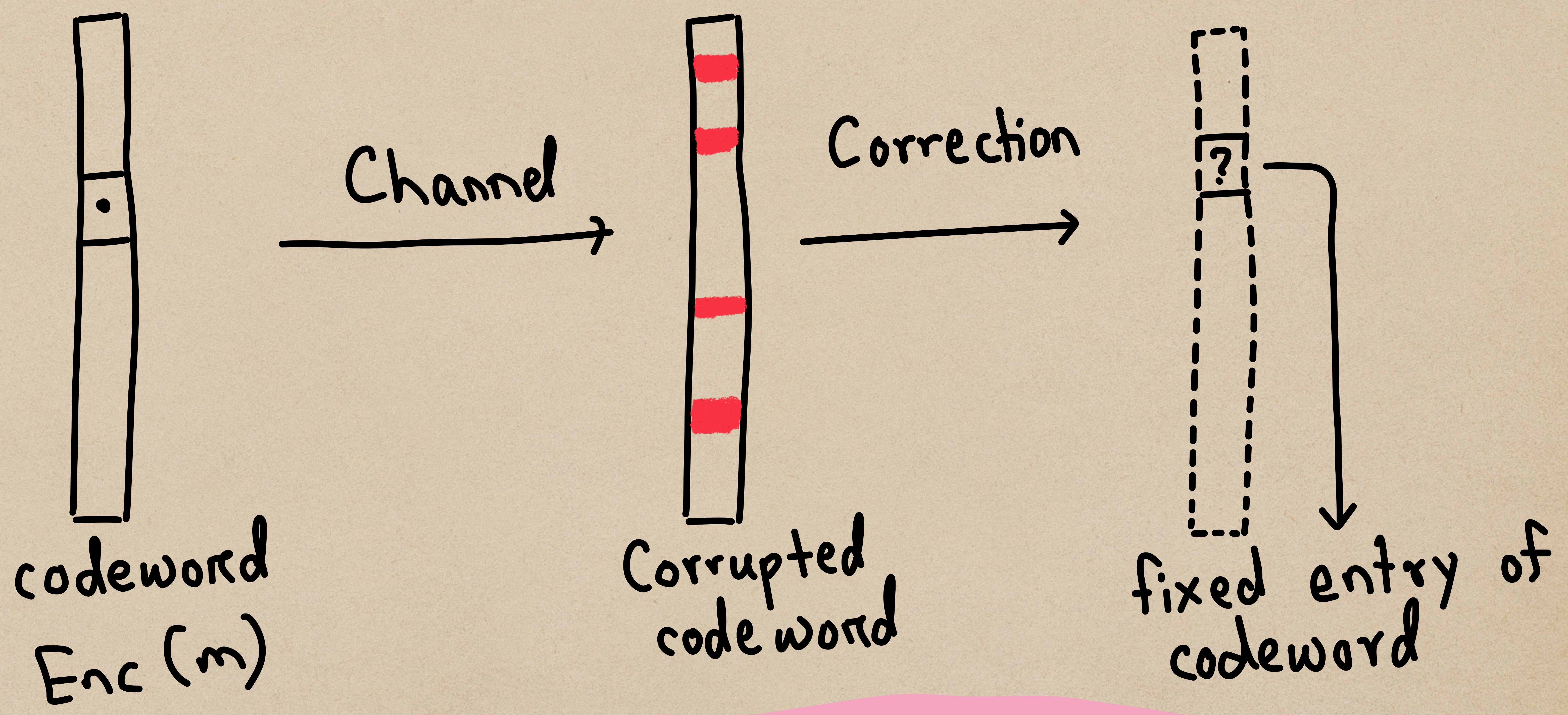
Error Correction : Given a corrupted codeword, Recover the original message

What is Error Correction ?

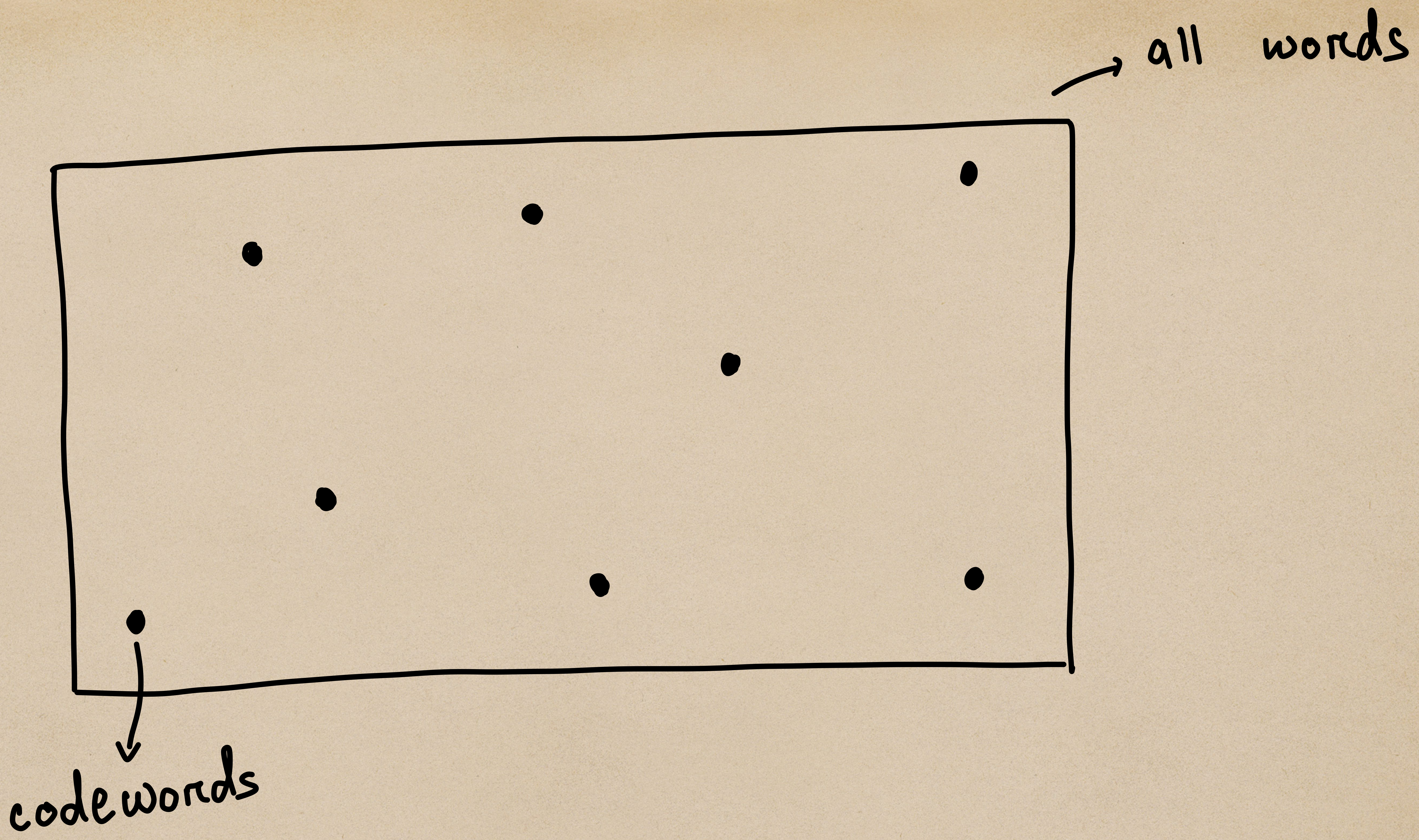


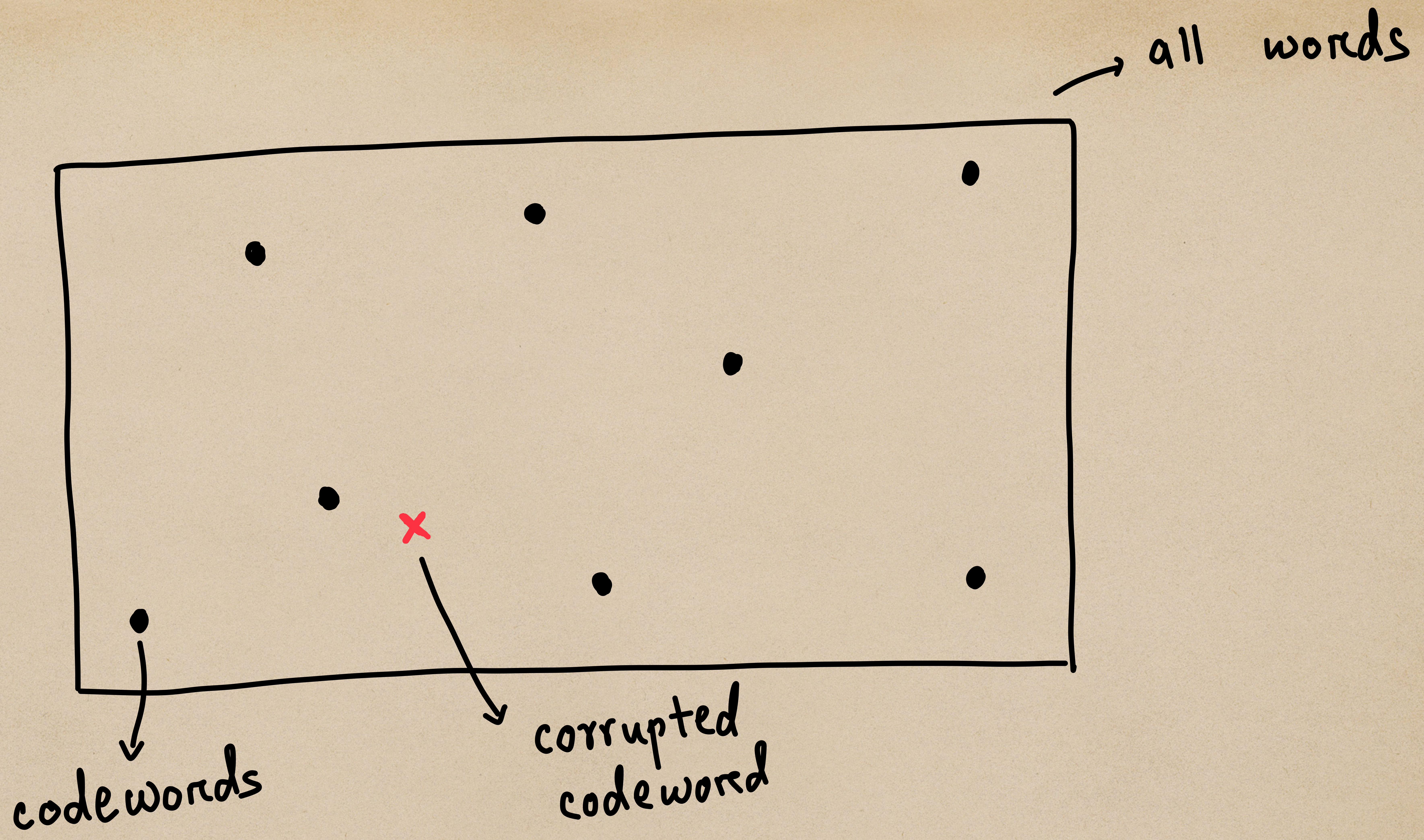
Error Correction : Given a corrupted codeword, Recover the original codeword

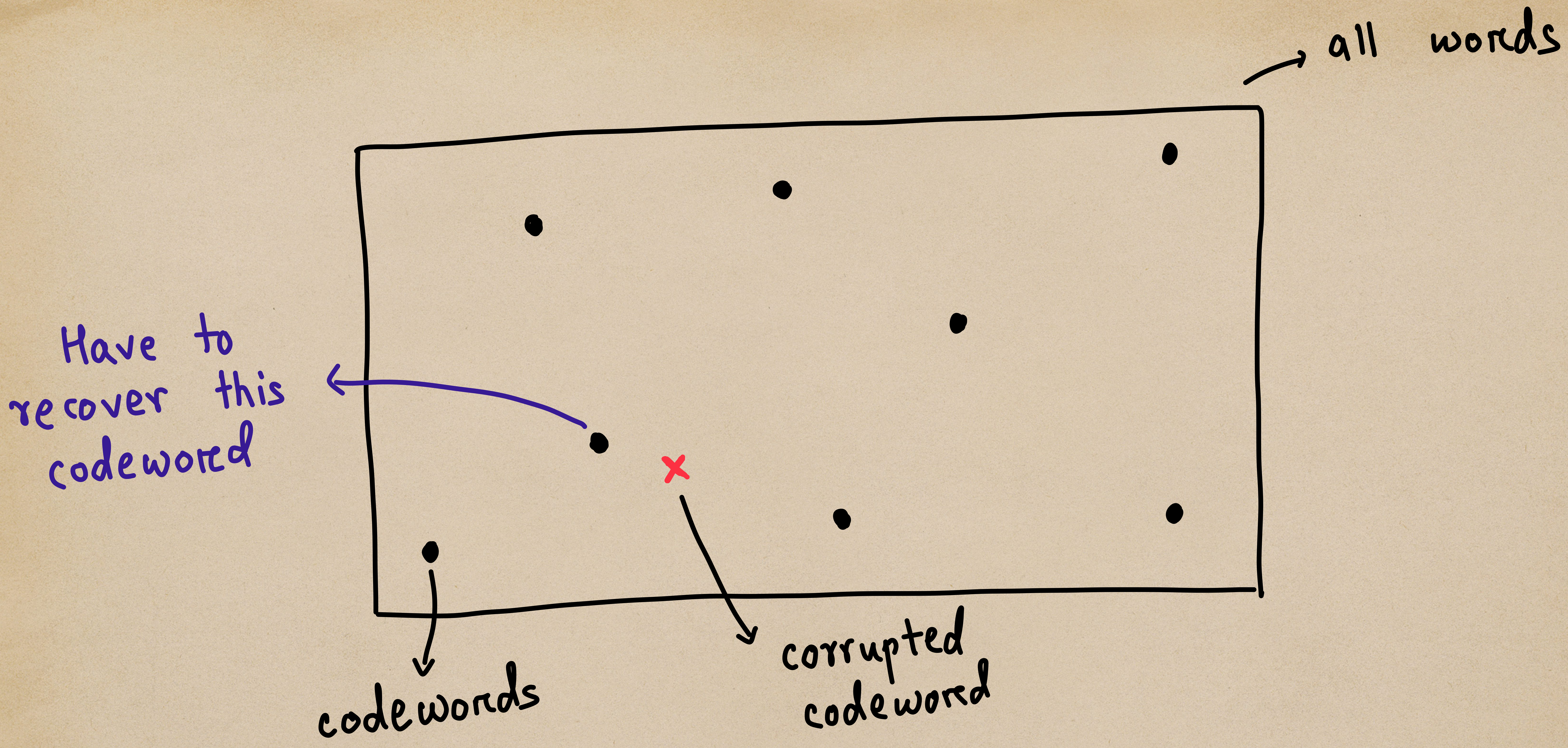
What is Local Correction ?

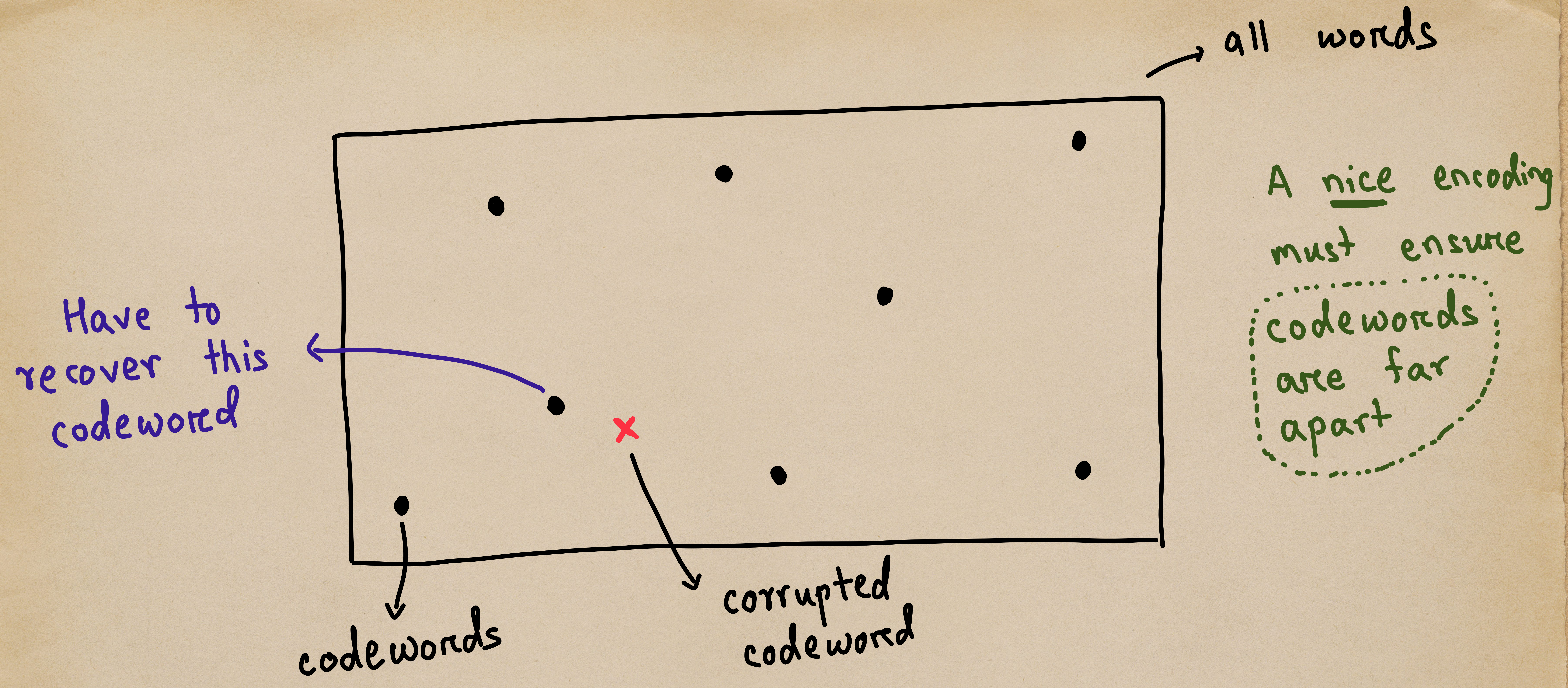


Local Correction : Given a corrupted codeword, Recover a fixed entry of the original codeword all words


all words
codewords


all words
codewords
corrupted codeword


all words
Have to recover this codeword
codewords
corrupted codeword


all words
A nice encoding must ensure codewords are far apart
Have to recover this codeword
codewords
corrupted codeword

Low-degree Polynomials as Error Correcting Codes

$P(x_1, x_2, \ldots, x_n) : \{0,1\}^n \longrightarrow \mathbb{R}$ — deg 1 poly.

E.g. $4x_1 - \frac{2}{3}x_2 + 5x_{10} - 99$

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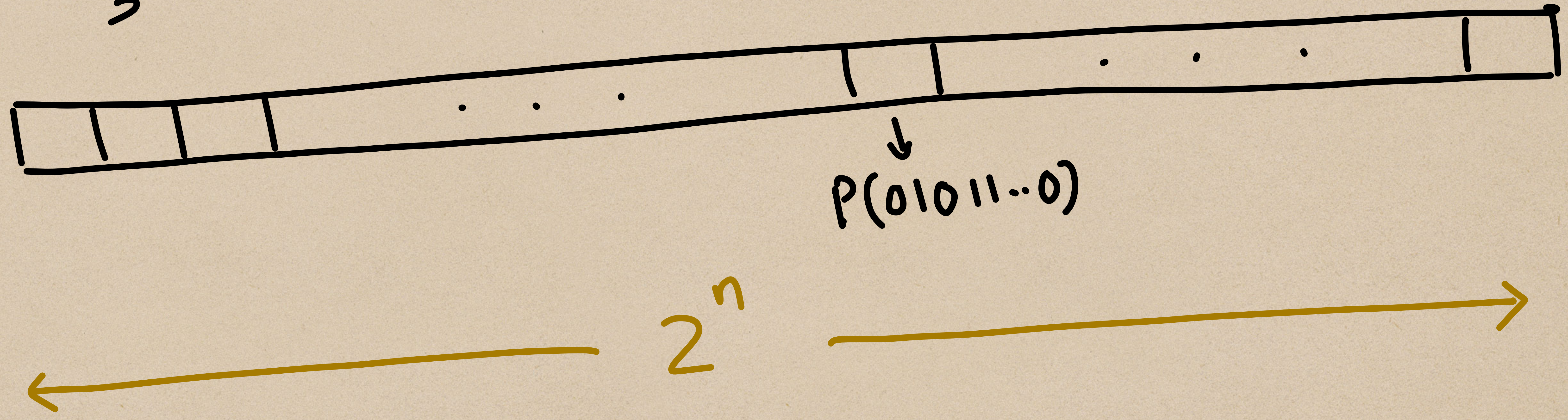
$P \equiv$



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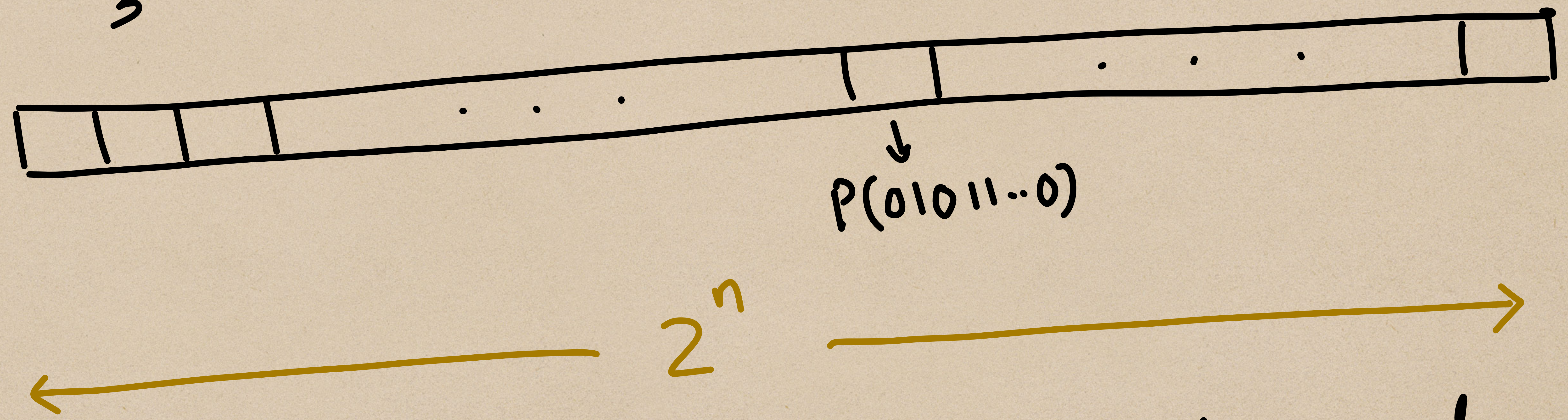

Codewords : Evaluation table of deg-1 polynomials over $\{0,1\}^n$

Low-degree Polynomials are Nice

Fact: Two distinct degree-1 polynomials on $\{0,1\}^n$ disagree on $\geq \frac{1}{2}$-fraction of $\{0,1\}^n$.

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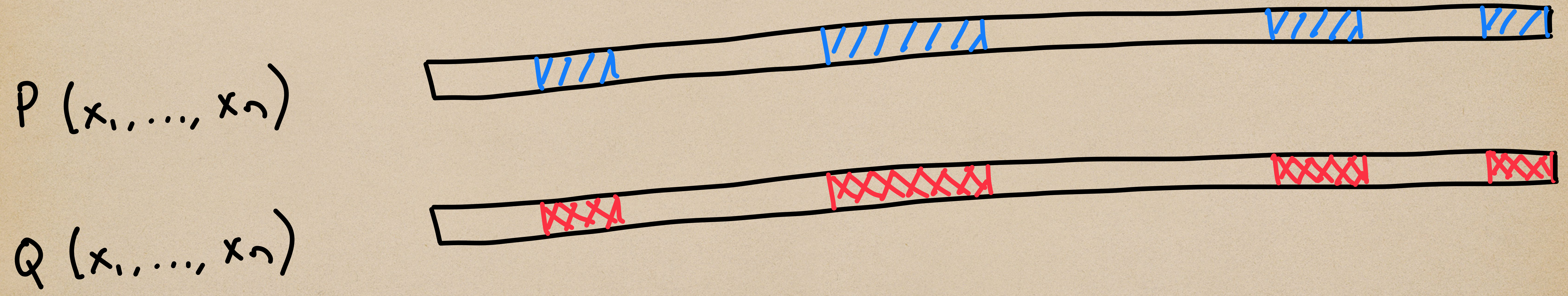
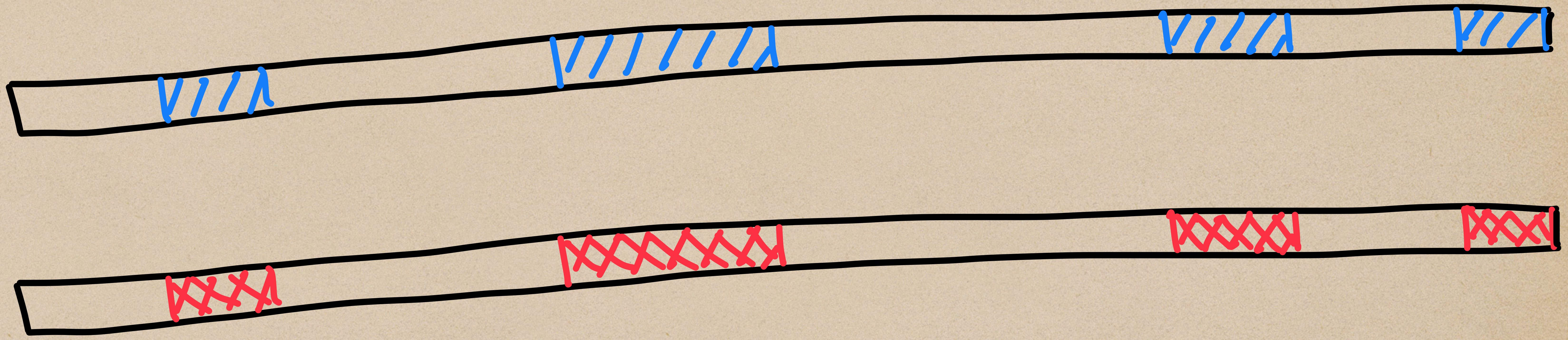


Low-degree Polynomials are Nice

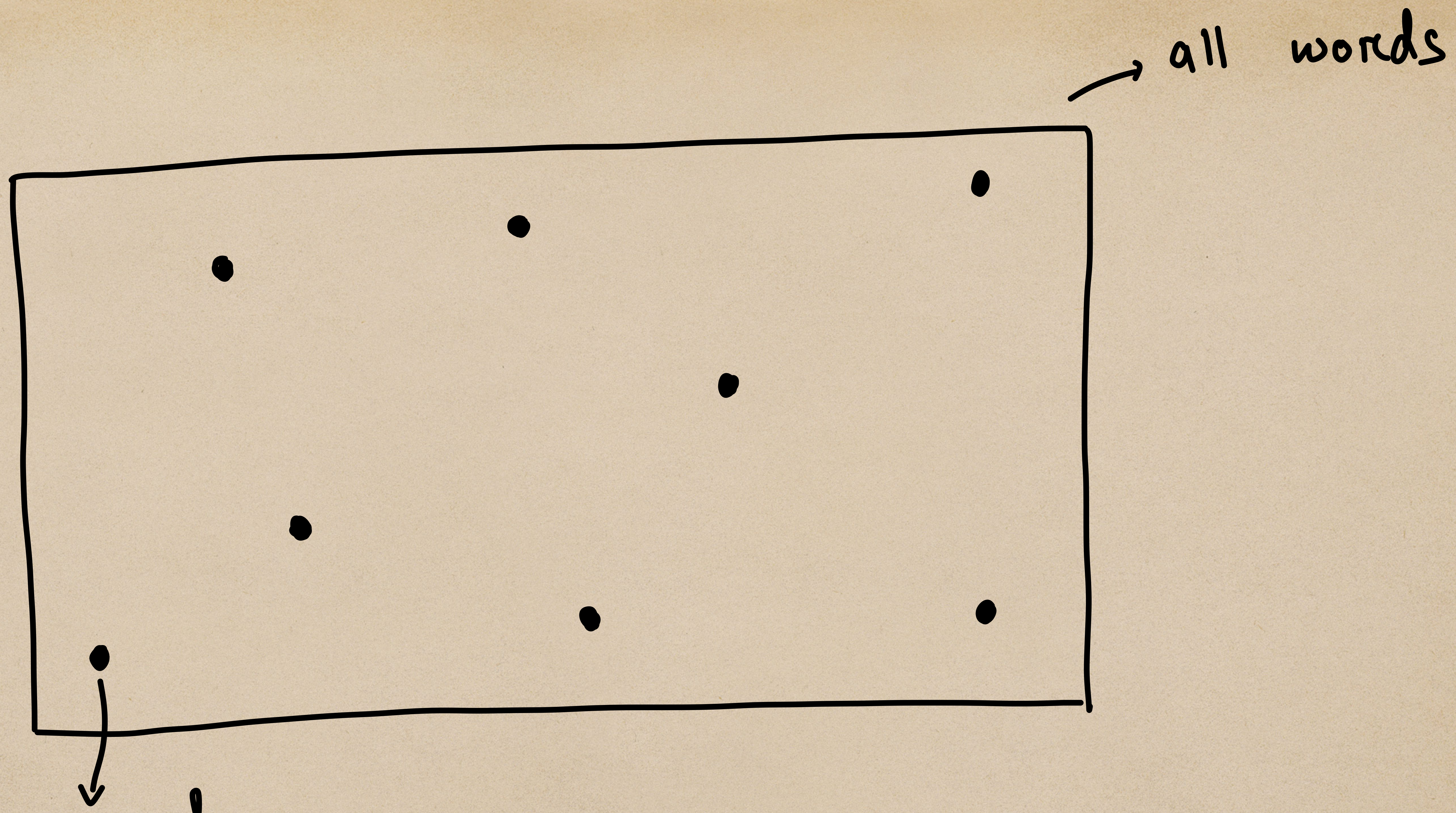
Fact: Two distinct degree-1 polynomials on $\{0,1\}^n$ disagree on $\geq \frac{1}{2}$-fraction of $\{0,1\}^n$.

$P(x_1, \dots, x_n)$

$Q(x_1, \dots, x_n)$

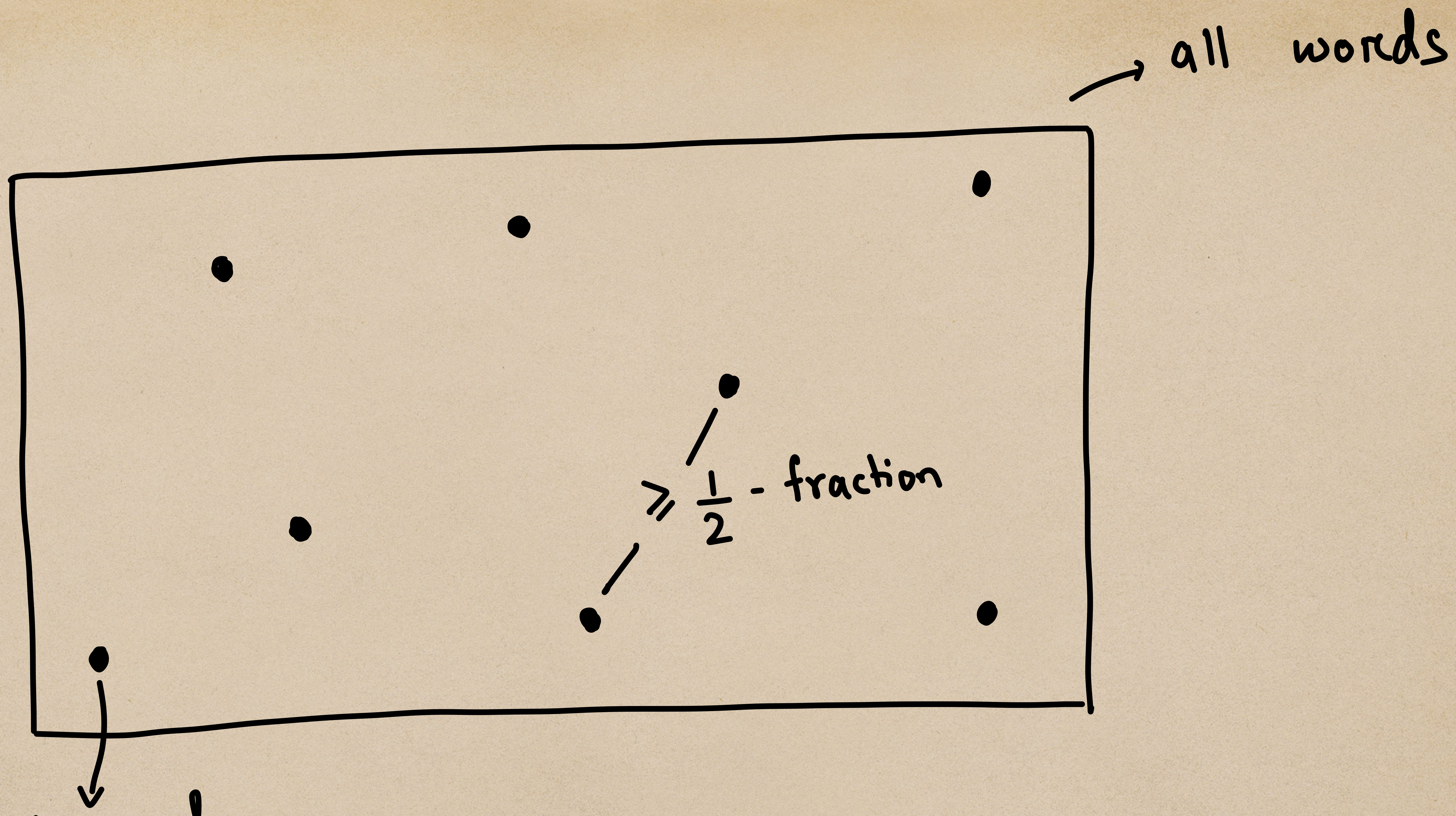


"If they disagree on a single point, they disagree on a lot of points."



codewords

= Evaluation of deg. 1 poly. on $\{0,1\}^n$



codewords
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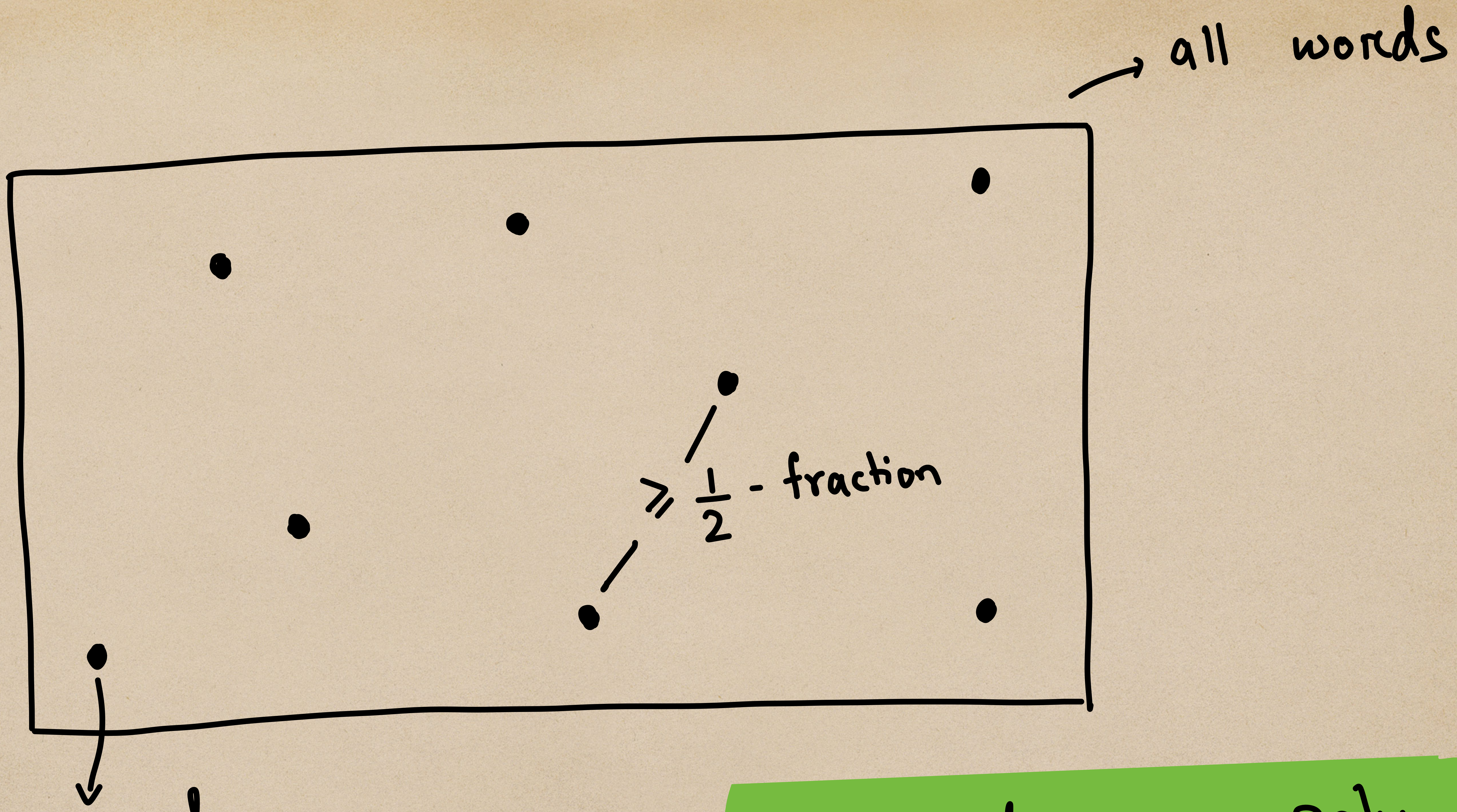
poly. on $\{0,1\}^n$



codewords
= Evaluation of deg. 1
poly. on $\{0,1\}^n$

Low-degree poly. are far apart.

Low-degree are Quite Useful

— Randomized Algorithms, e.g. matching

— Probabilistically Checkable Proofs (PCPs)

— Combinatorics, e.g. Kakeya's conjecture, Polynomial Method

⋮

Correction Algorithm for Low-degree Polynomials

Input $\quad f : \{0,1\}^n \longrightarrow \mathbb{R}$

$f \equiv$ 

Correction Algorithm for Low-degree Polynomials

Input $f : \{0,1\}^n \to \mathbb{R}$

Corruption at unknown places

$f \equiv$

$p \equiv$

p → degree-1 poly.

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Input $f : \{0,1\}^n \longrightarrow \mathbb{R}$

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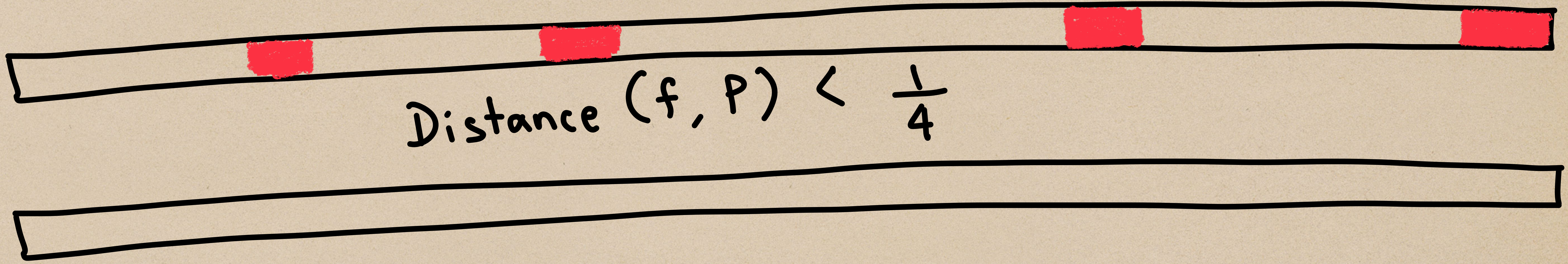

$P \equiv$

$\downarrow$

degree-1 poly.

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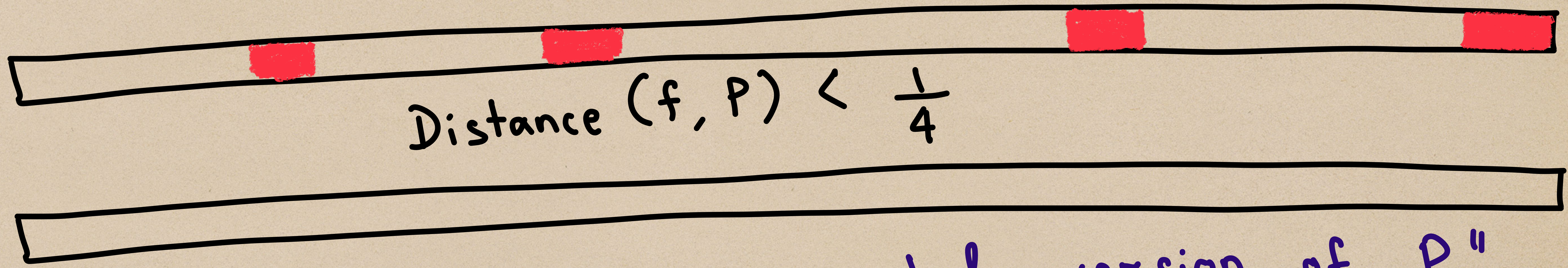


$\text{Distance}(f, P) < \frac{1}{4}$

$P \equiv$

"f is a corrupted version of P"

degree-1 poly.

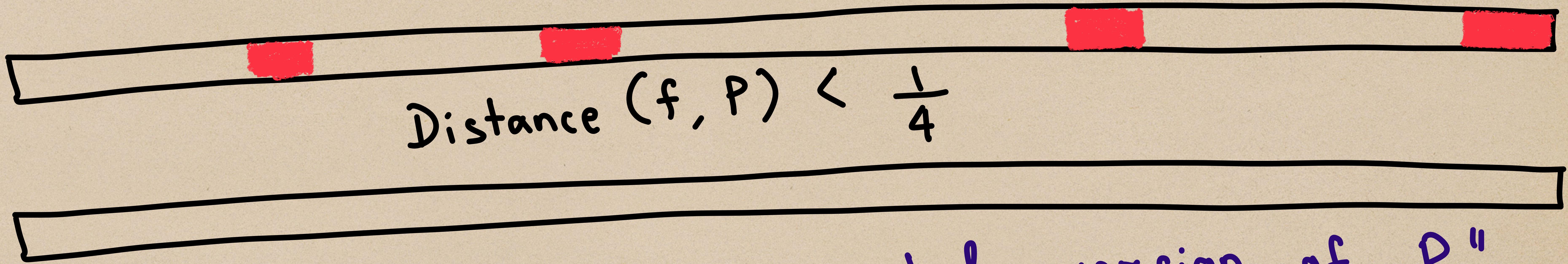
Correction Algorithm for Low-degree Polynomials

Input $f: \{0,1\}^n \to \mathbb{R}$

Corruption at unknown places

$f \equiv$

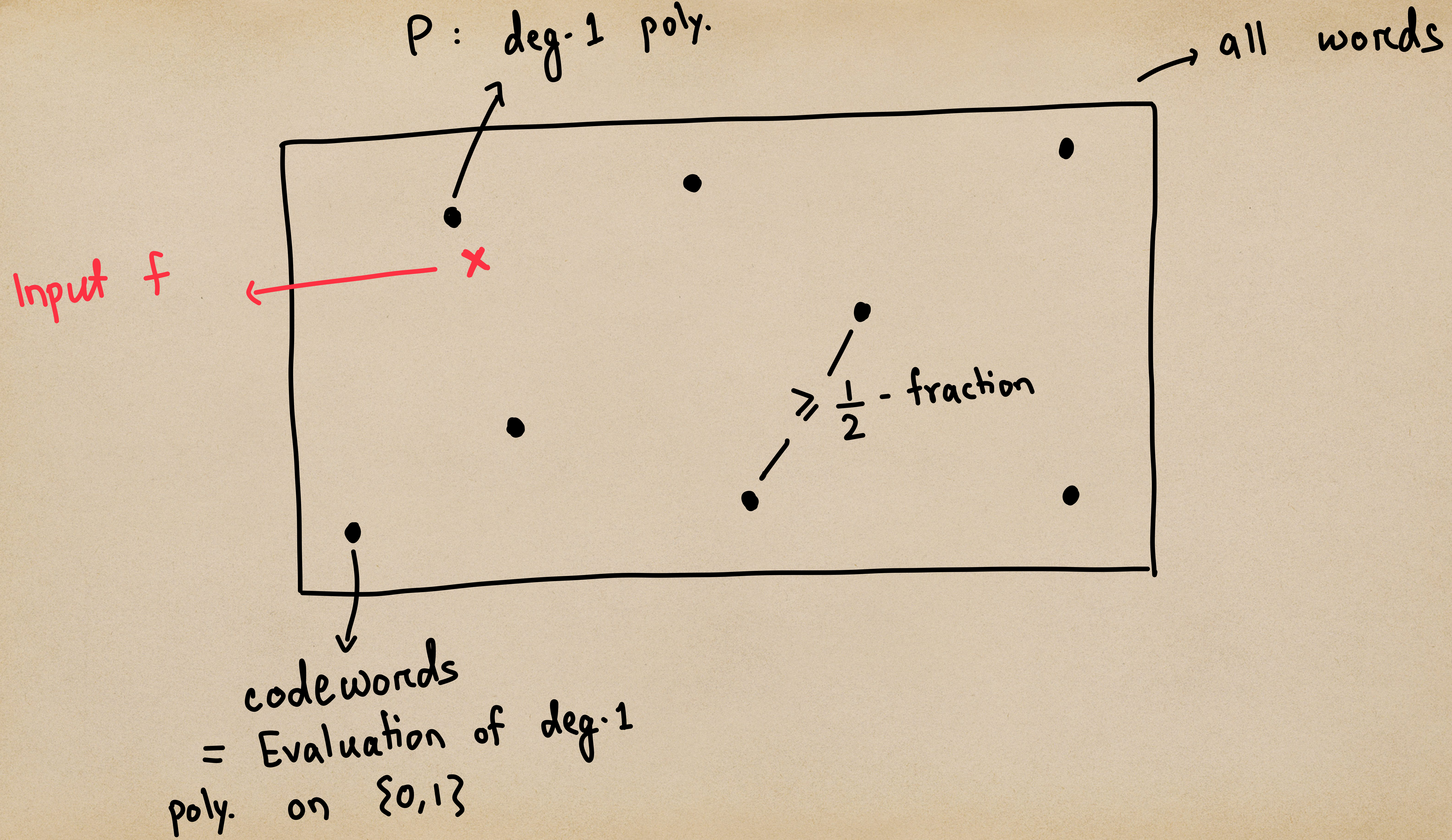
$P \equiv$



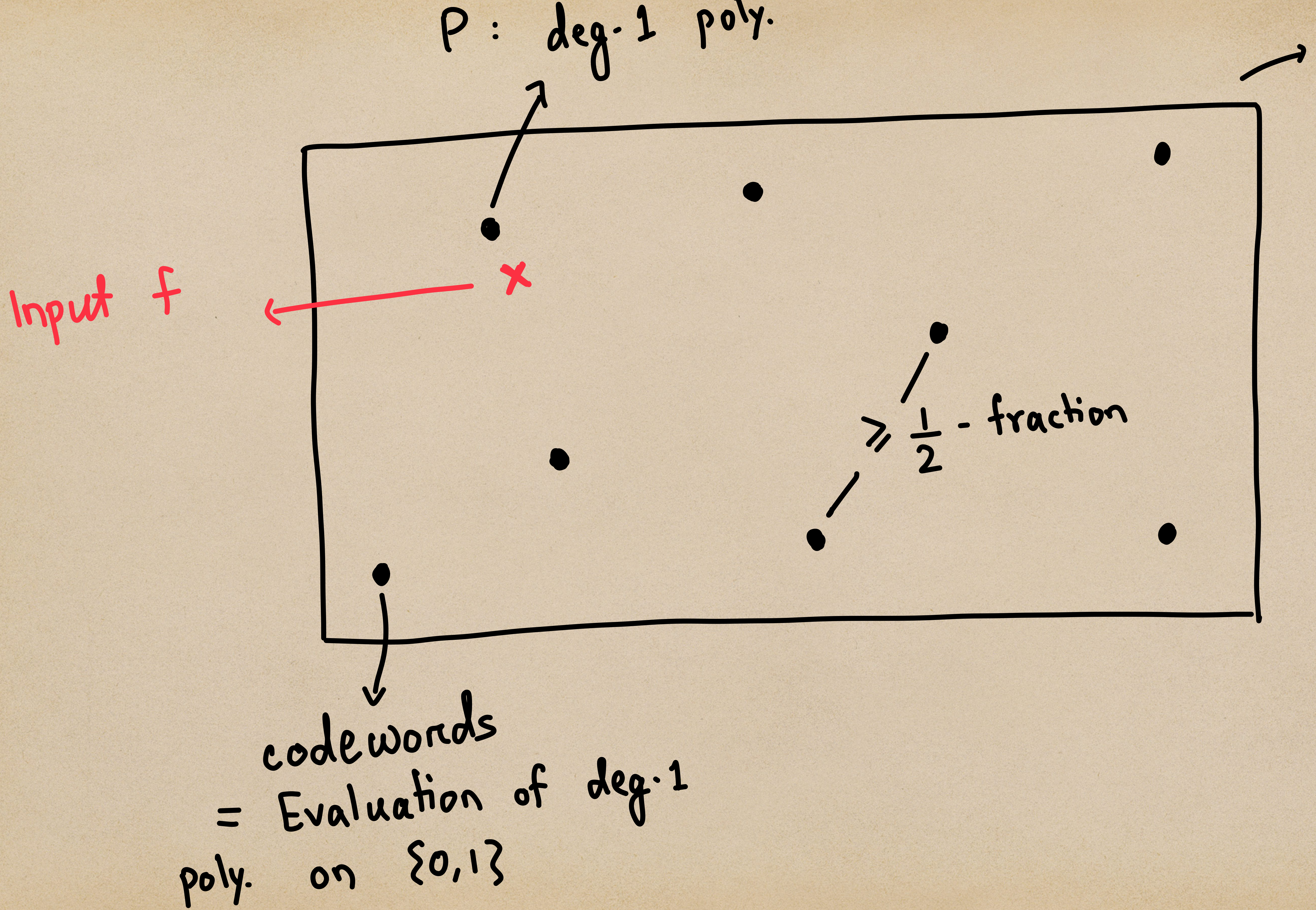
"f is a corrupted version of P"

P → degree-1 poly.

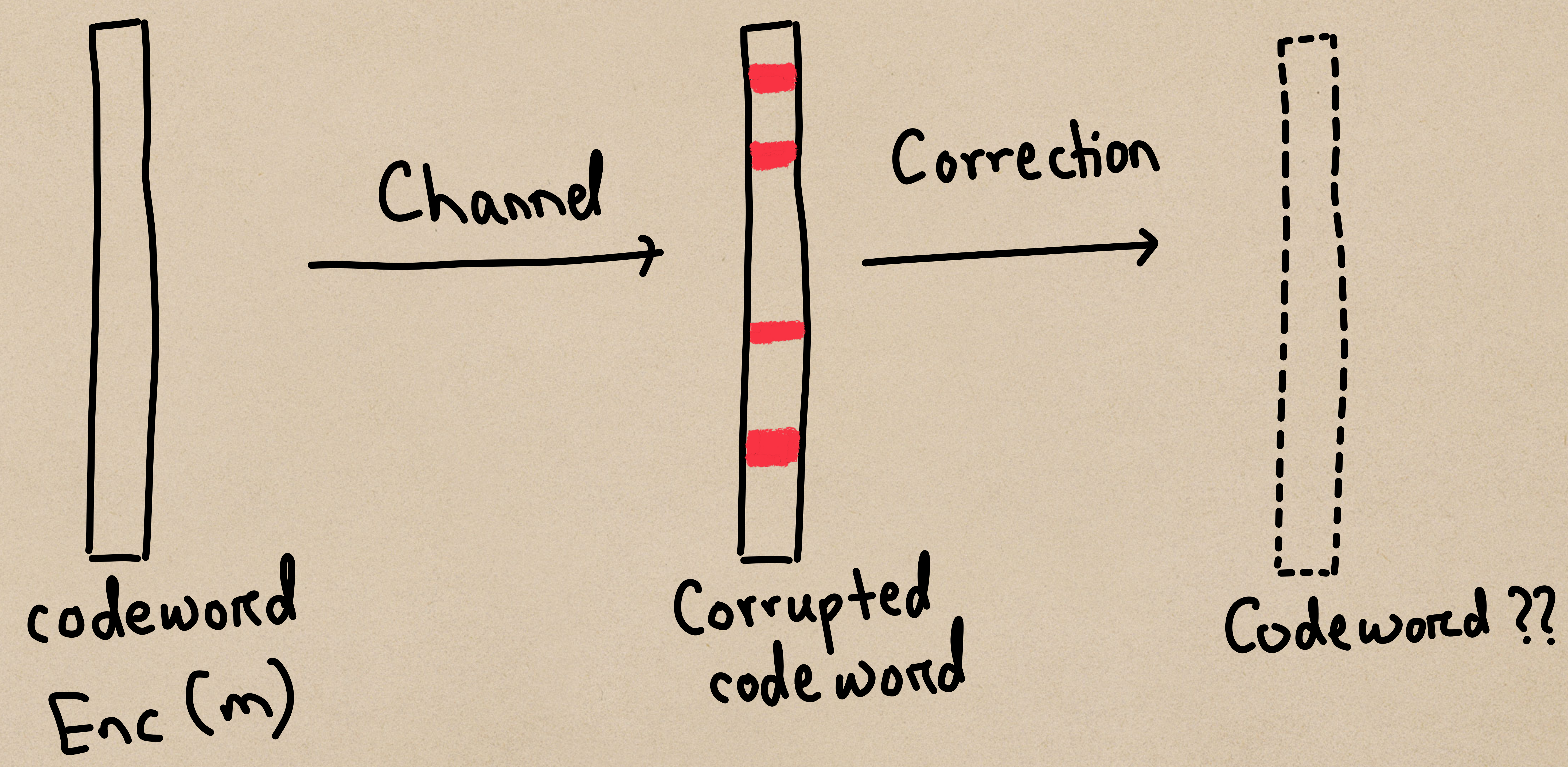
For input $f(x_1, \ldots, x_n)$, deg-1 poly. $P(x_1, \ldots, x_n)$ is unique bcoz of distance of deg-1 polynomials.

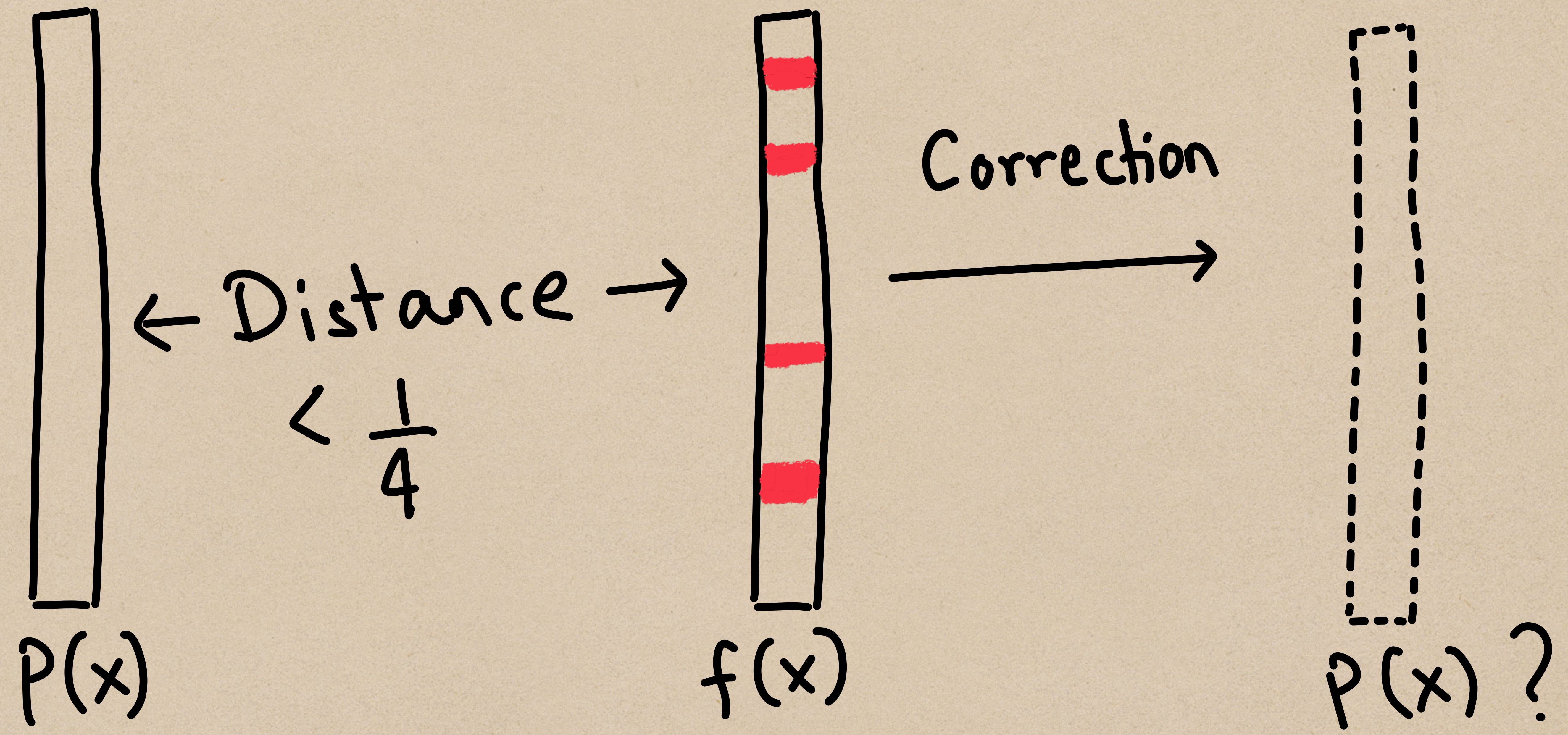


P : deg. 1 poly.
all words
Input f
$\geq \frac{1}{2}$ - fraction
codewords
= Evaluation of deg. 1
poly. on {0,1}



Given access to $f(x)$, how efficiently can we recover $P(x)$?

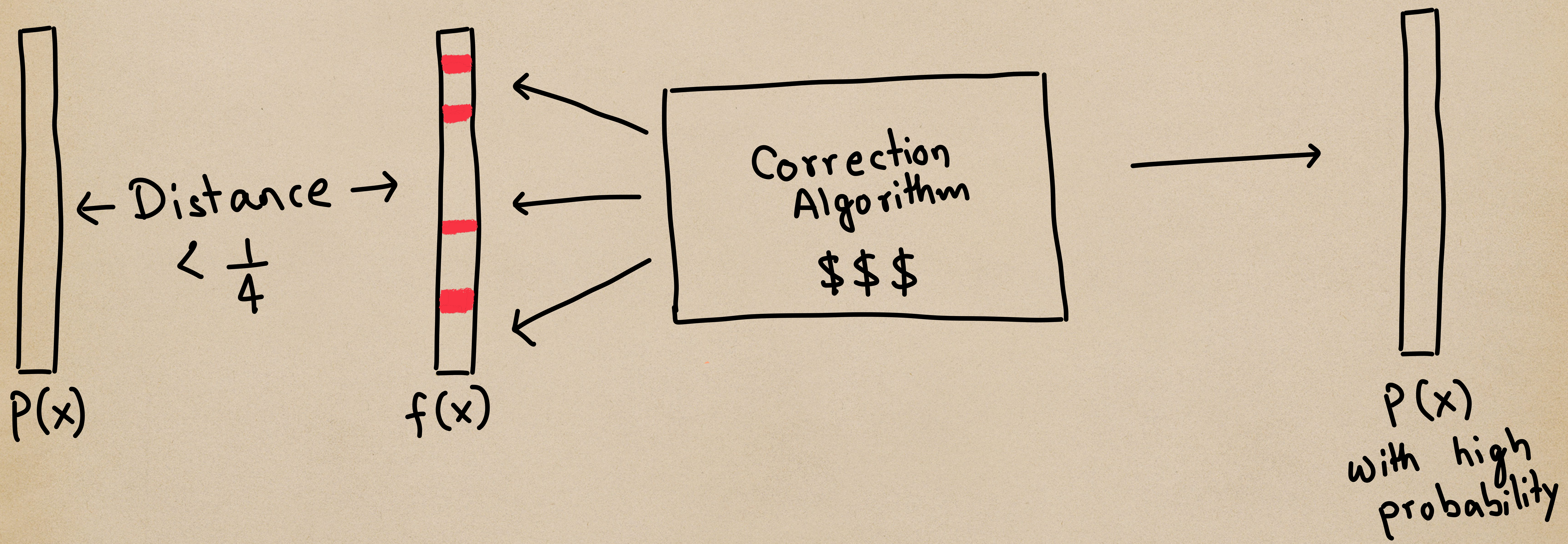

Channel
Correction
codeword
Enc (m)
Corrupted
code word
Codeword ??



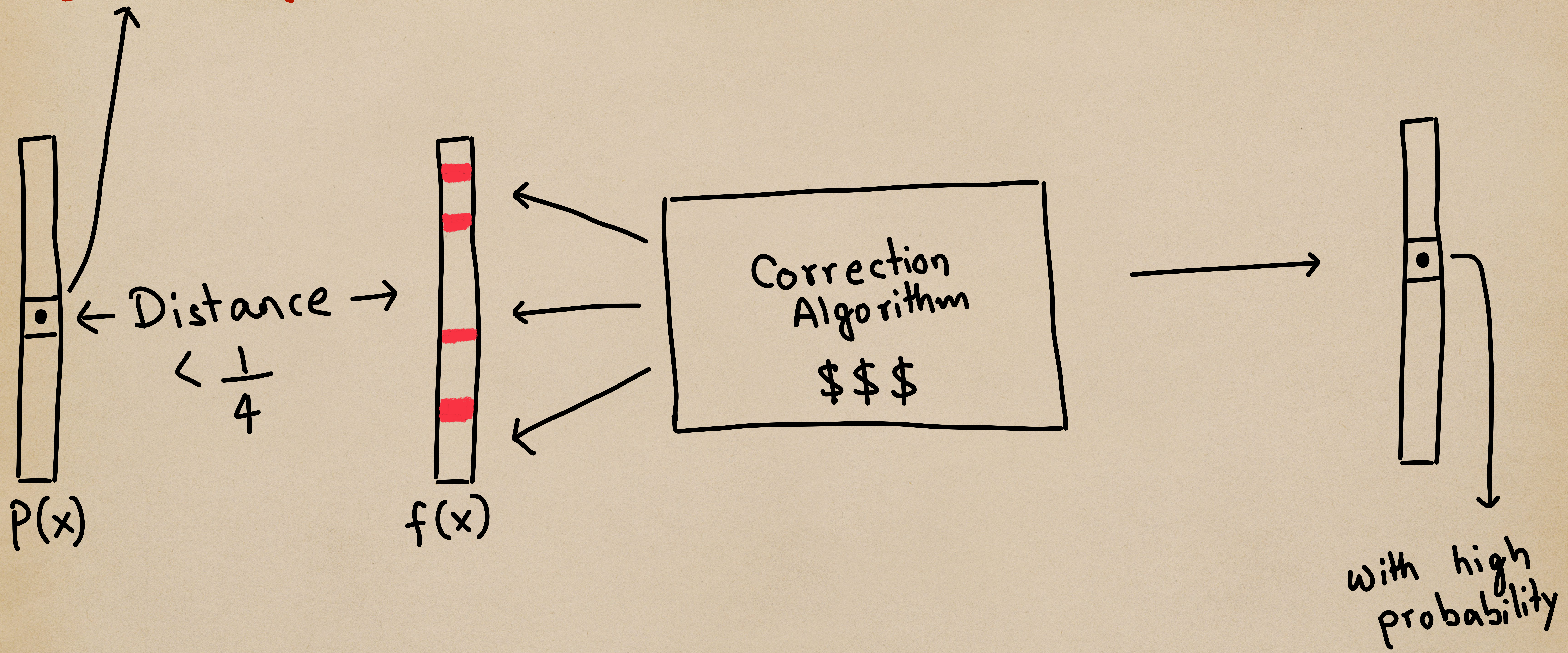
Distance
$< \frac{1}{4}$
Correction
$P(x)$
$f(x)$
$P(x)$?

Correction for Low-degree Polynomials

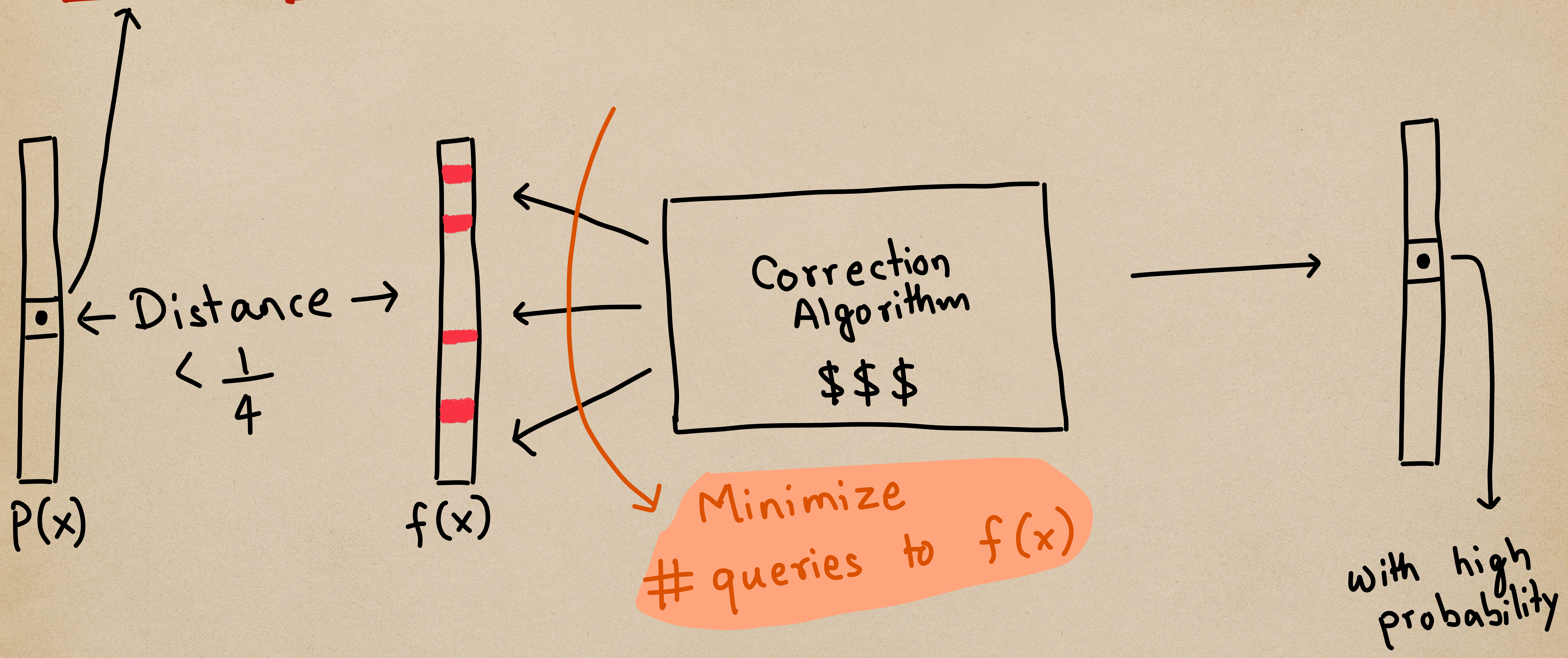
Correction for Low-degree Polynomials



Local Correction for Low-degree Polynomials



Local Correction for Low-degree Polynomials



Prior Work

- [Goldreich - Levin '89] : Linear functions $\mathbb{F}_2^n \longrightarrow \mathbb{F}_2$

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- [Beaver- Feigenbaum '90], [GLRSW '91], [Gemell- Sudan '92], [Sudan- Trevisan- Vadhan '01], [Gopalan- Klivans - Zuckerman '08], [Bhowmick - Lovett '18], ...

  Low - degree polynomials $\mathbb{F}_p^n \to \mathbb{F}_p$

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  Low - degree polynomials $\mathbb{F}_p^n \to \mathbb{F}_p$

- Useful in
  - Cryptography (One - way functions)
  - Pseudorandomness (Hardness Amplification)

Our Setting

- [Kim - Kopparty '17] : Correction for low-degree polynomials in poly(n) queries

Our Setting

- [Kim-Kopparty '17]: Correction for low-degree polynomials in poly(n) queries

- [Bafna-Srinivasan-Sudan '20]: Local correction for low-degree polynomials

$$P(x) : \{0,1\}^n \to \mathbb{F}_q$$

(constant characteristic)

in $O(1)$ queries.

Our Results

$f(x) : \{0,1\}^n \longrightarrow \mathbb{R}$ $\qquad$ Distance $(f, P) < \frac{1}{4}$

(degree 1) $\qquad P(x) : \{0,1\}^n \longrightarrow \mathbb{R}$

Our Results

$f(x) : \{0,1\}^n \longrightarrow \mathbb{R}$ Distance $(f, P) < \frac{1}{4}$

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ABPSS'25: Degree-1 polynomials can be locally corrected using nearly $\tilde{O}(\lg n)$ queries.

Our Results

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ABPSS'25: Degree-1 polynomials can be locally corrected using nearly $\tilde{O}(\lg n)$ queries.

- Nearly optimal because of a lower bound by [Bafna, Srinivasan, and Sudan' 20].

Our Results

$f(x) : \{0,1\}^n \longrightarrow \mathbb{R}$ Distance$(f, P) < \frac{1}{2^{d+1}}$

(degree d) $P(x) : \{0,1\}^n \longrightarrow \mathbb{R}$

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We also have combinatorial list decoding bound and sublinear query local list correction algorithm.

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THANK YOU !!

Questions?
Comments?