Redistricting : electoral maps in France for parliamentary elections

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[1] Découpage électoral des circonscriptions législatives en France: déséquilibres démographiques et contraintes territoriales.
T. Ehrhard,
S. Attias,
E. Bampis,
V. Cohen-Addad,
B. Escoffier,
C. Mathieu,
F. Pascual,
A. Pass-Lanneau,
D. Saulpic.
Revue Française de Science Politique 2022

[2] Estimating the Electoral Consequences of Legislative Redistricting in France.
T. Ehrhard,
E. Bampis,
B. Escoffier,
C. Mathieu,
F. Pascual,
D. Saulpic.
ACM FAccT 2025.

Slides partially from Bruno Escoffier and David Saulpic

Parliamentary (legislative) elections in France

Assemblée nationale

This procedure :

- Provides a clear majority in the parliament
- Government from the largest group
- Election every 5 years

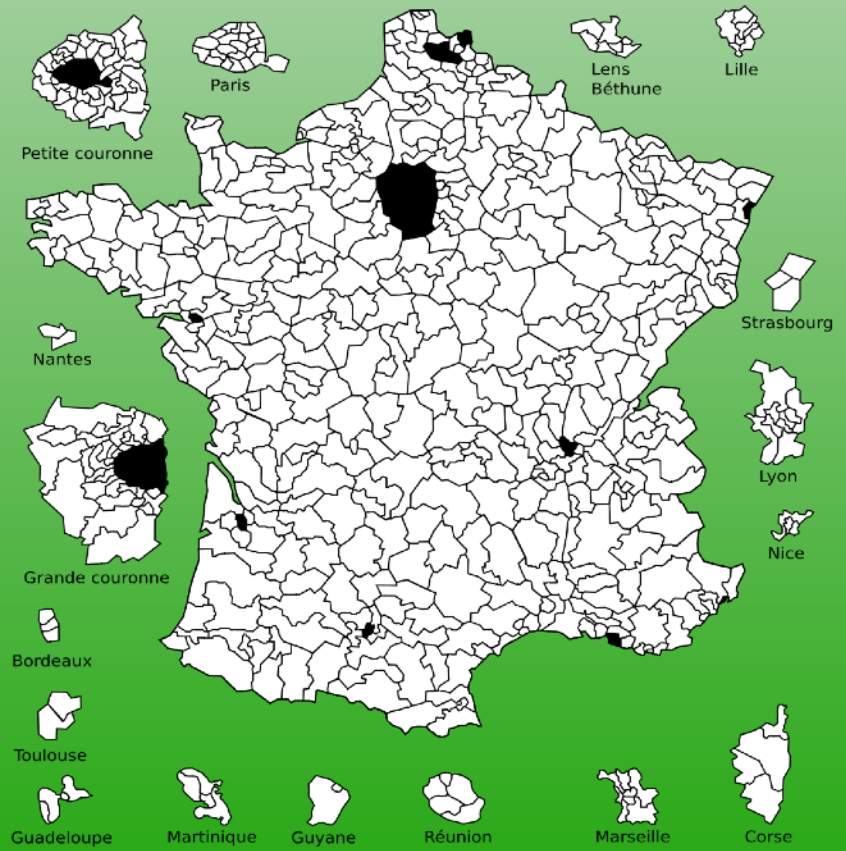
Parliamentary (legislative) elections in France

Assemblée nationale

How are Parliament Members (PM) elected ?

- 577 MP
- Territory divided into 577 districts (*circonscriptions*)
- In each district : one MP is elected through a two-round majority election

may have a strong impact on the result of the election !



Electoral map(s)

How to build this map ?

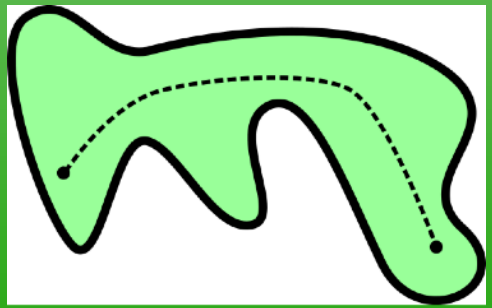
→ Must respect some constraints.

(I) Demographic balance

→ goal : each vote counts the same

(II) Connectivity

→ Each district is made of « one piece »

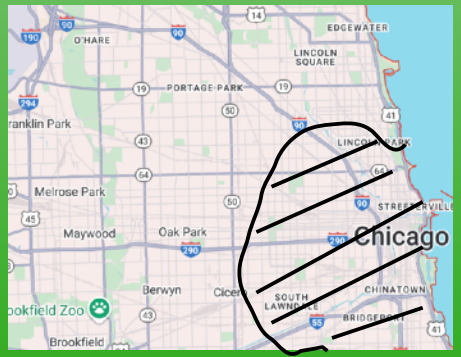


(III) Locality

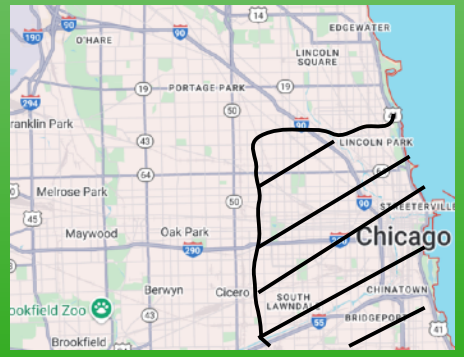
→ districts are inside *départements*

Electoral map(s)

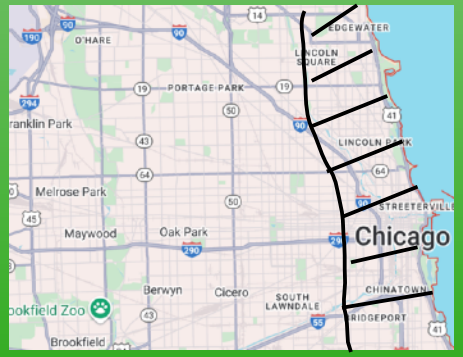
QUIZZ #1 : given these constraints, how would you build the district around Chicago ?



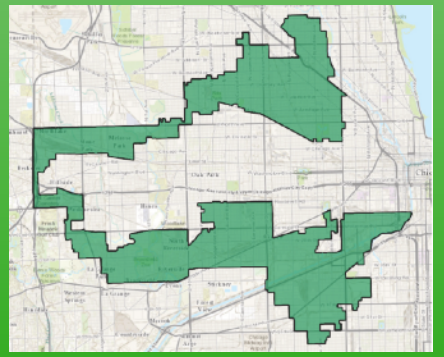
Answer A



Answer B



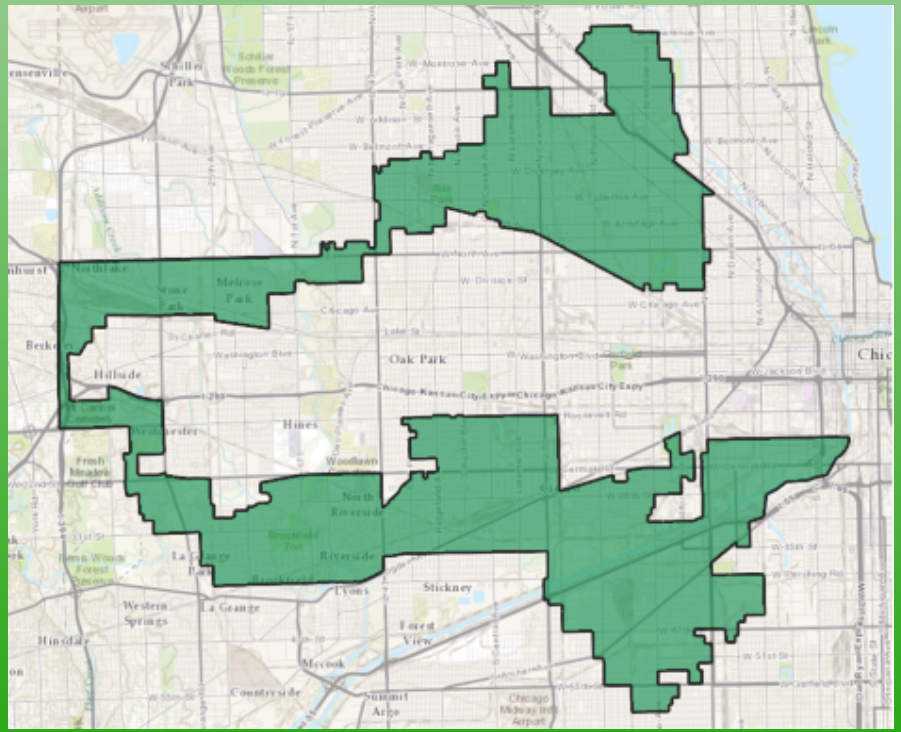
Answer C



Answer D

Electoral maps

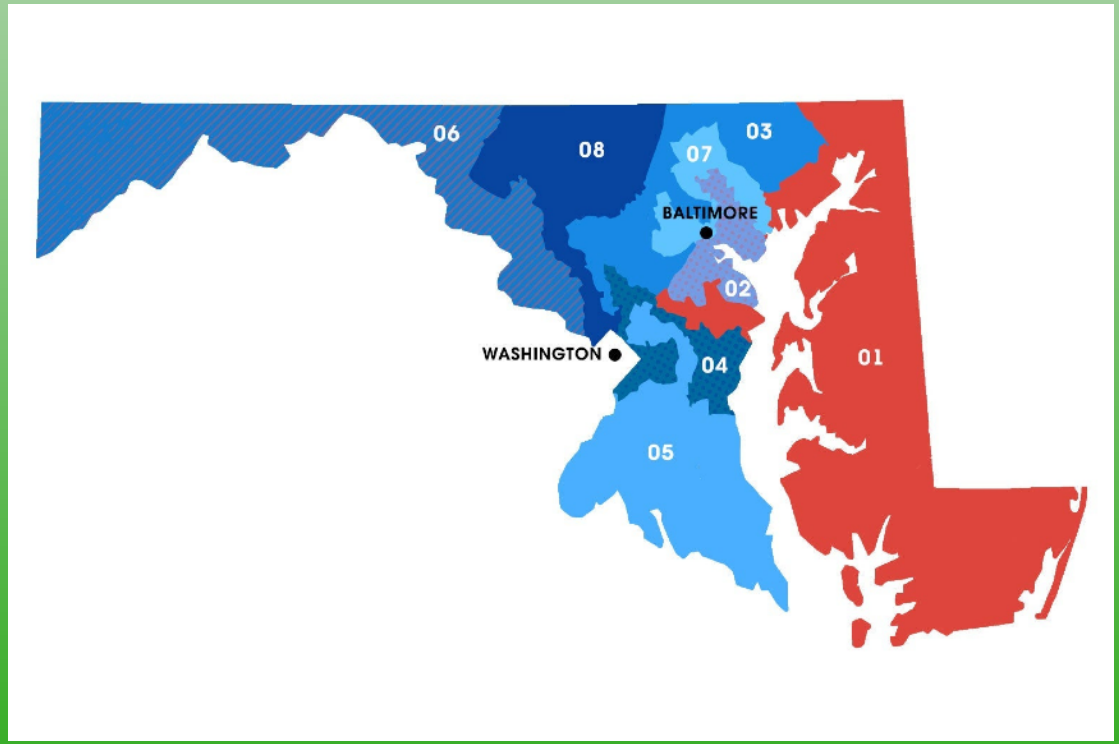
QUIZZ #1 : given these constraints, how would you build the district around Chicago ?



Answer D, of course !

Gerrymandering : the art of drawing district to favor one's own party

Gerrymandering

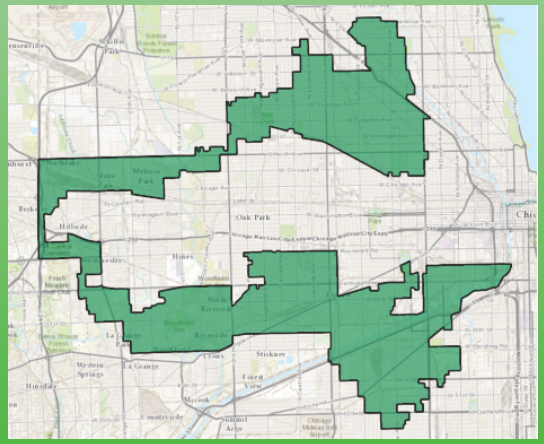


Slate: “Democrats Could Have Gerrymandered Away a GOP Seat. Why Didn’t They?”

Raskin said: Democrats can’t just be expected to sit quietly like a “Quaker meeting house” while Republicans are gerrymandering to their advantage everywhere.

Electoral map(s) in France

QUIZZ #2

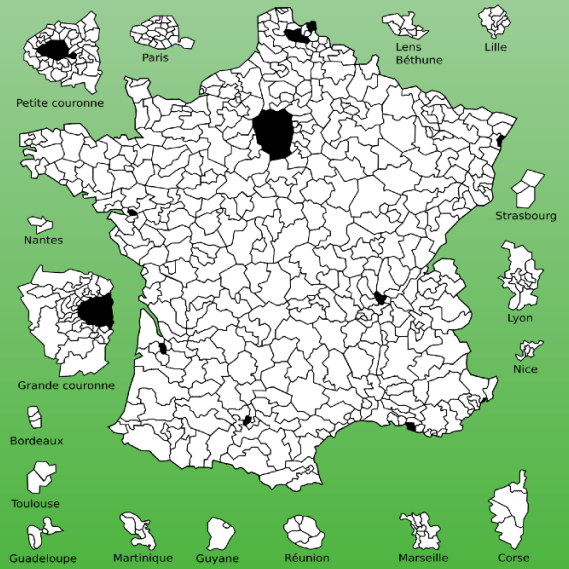


No such strange district in France.

Why not ?

(A) Because our politicians have much more integrity and will never gerrymander.

(B) Because there is a rule that somehow prevents it.



Electoral map(s)

QUIZZ #2

Cantons of the département #25 (Doubs)

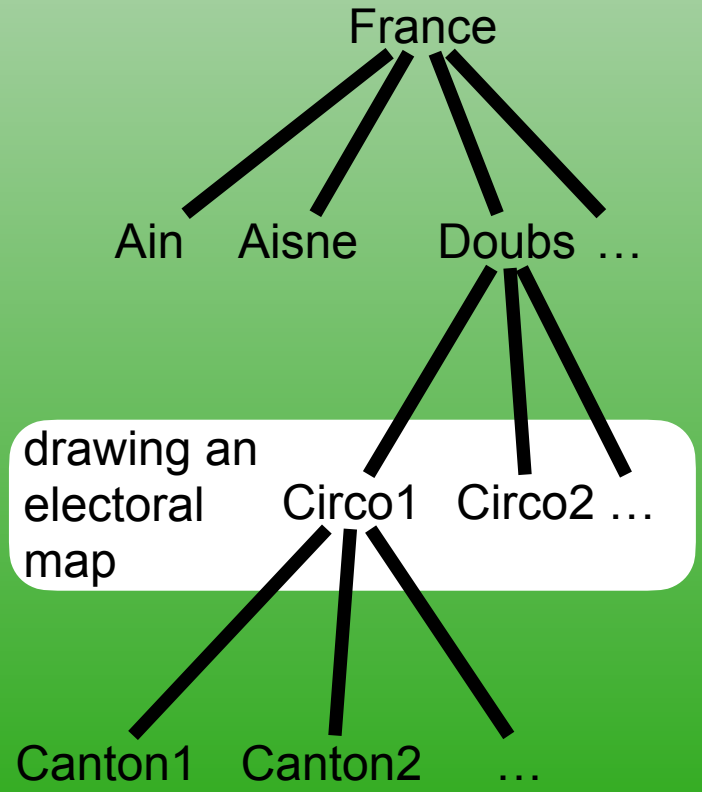
No such strange district in France.

Why not ?

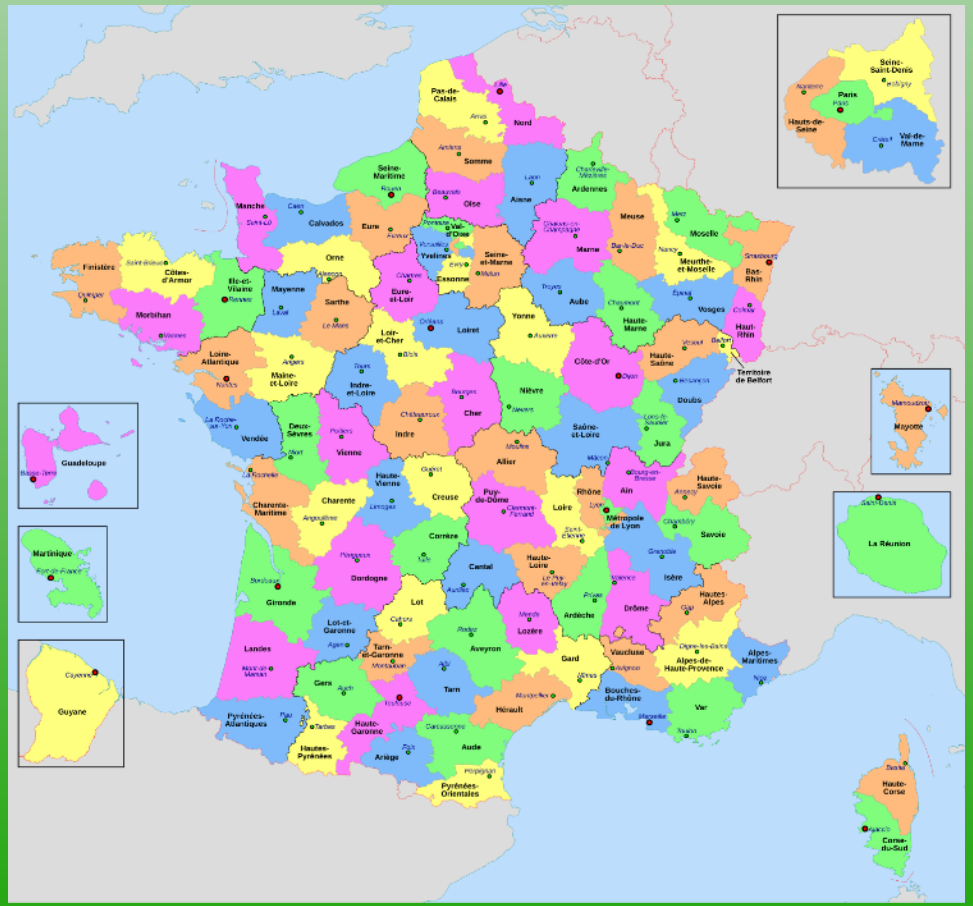
(A) Because our politicians have much more integrity and will never gerrymander.

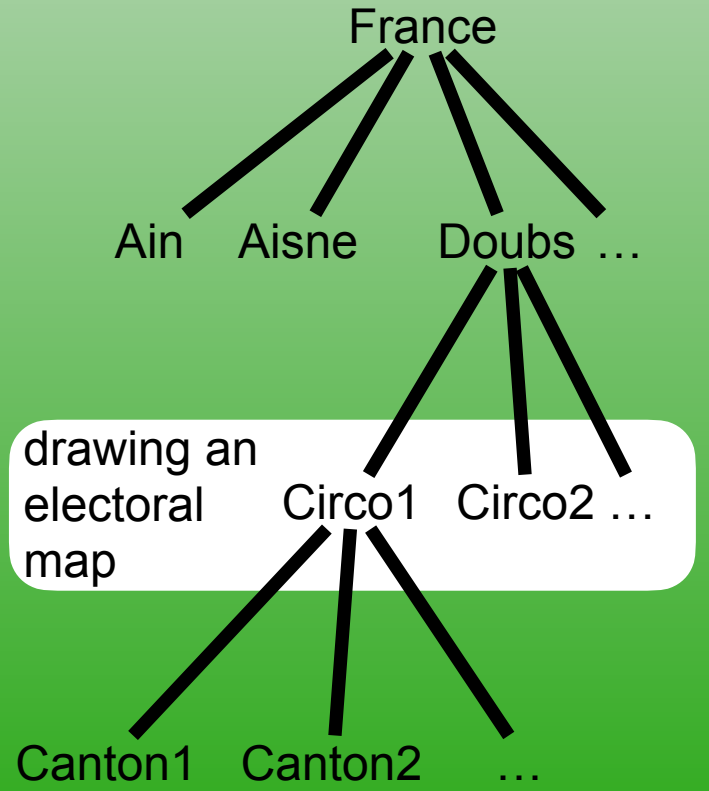
(B) Because there is a rule that somehow prevents it.

Rule : a département is partitioned into *cantons* and cantons should not be split into several circonscriptions (districts).

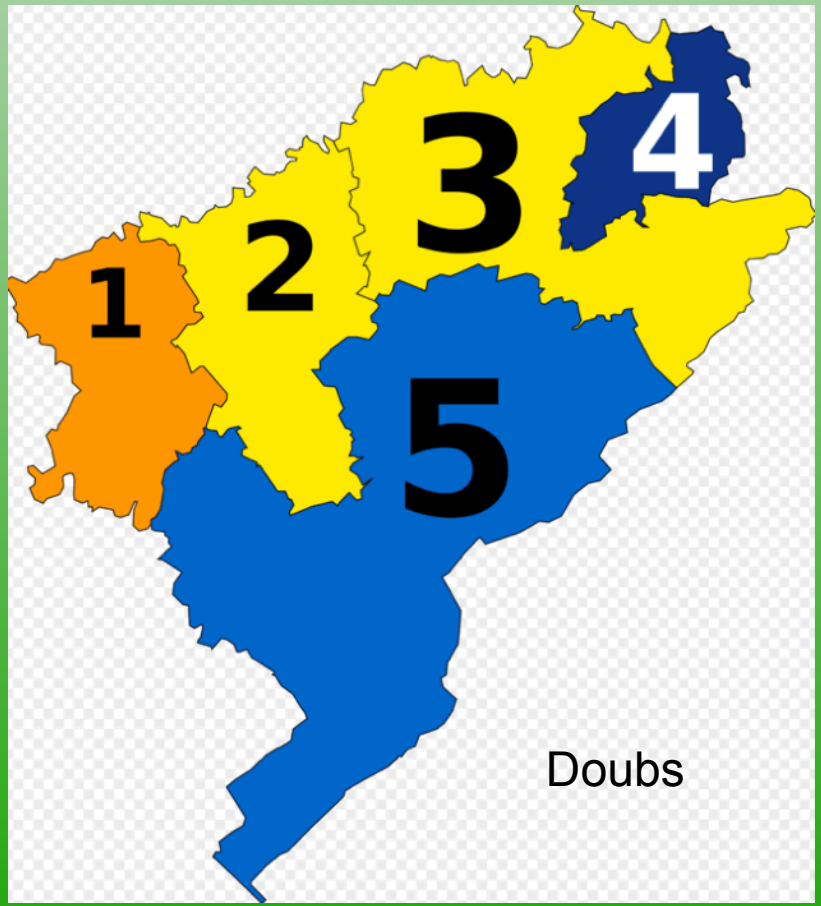


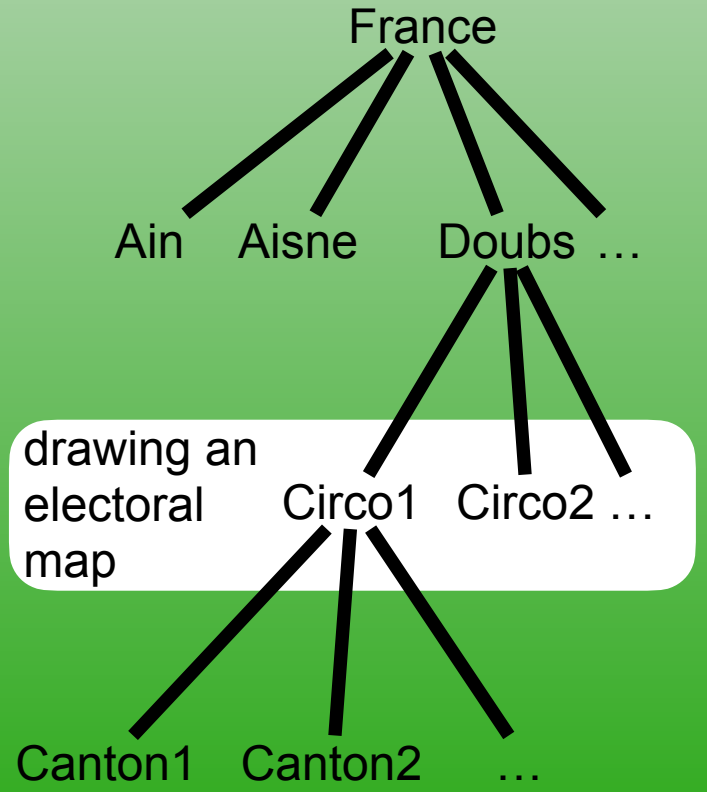
France is partitioned into 100 départements



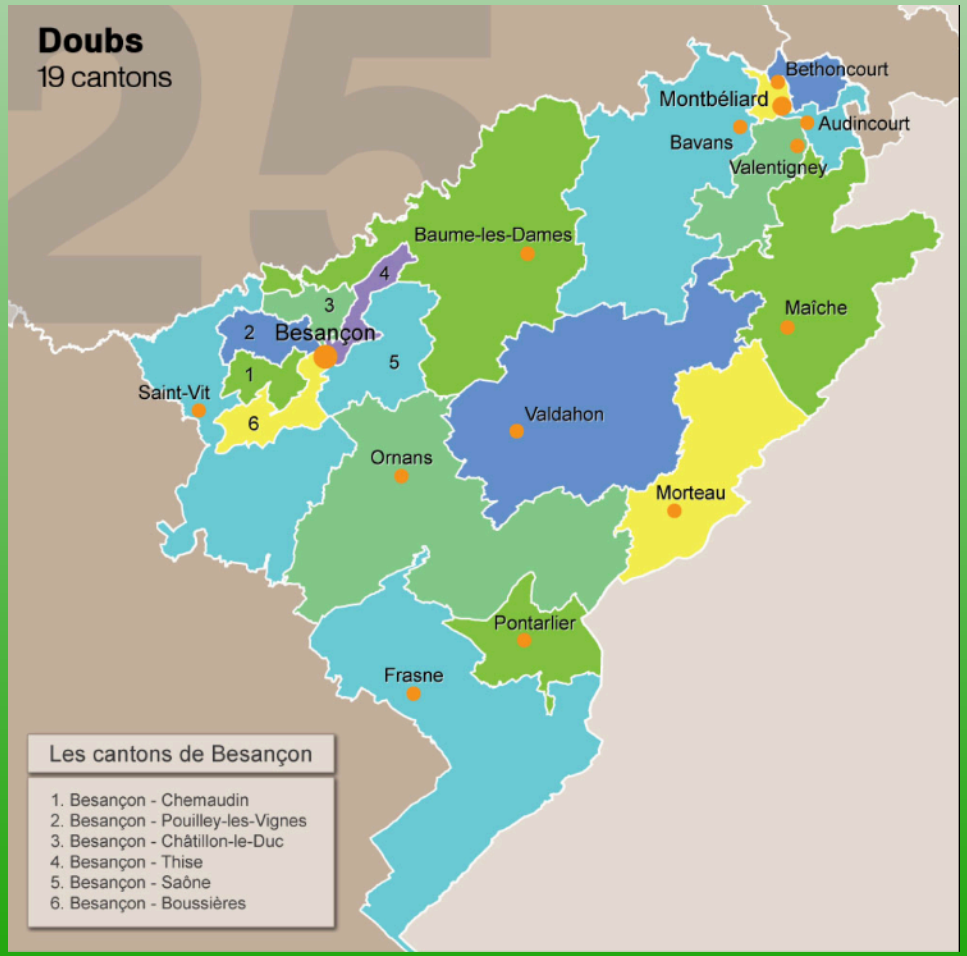


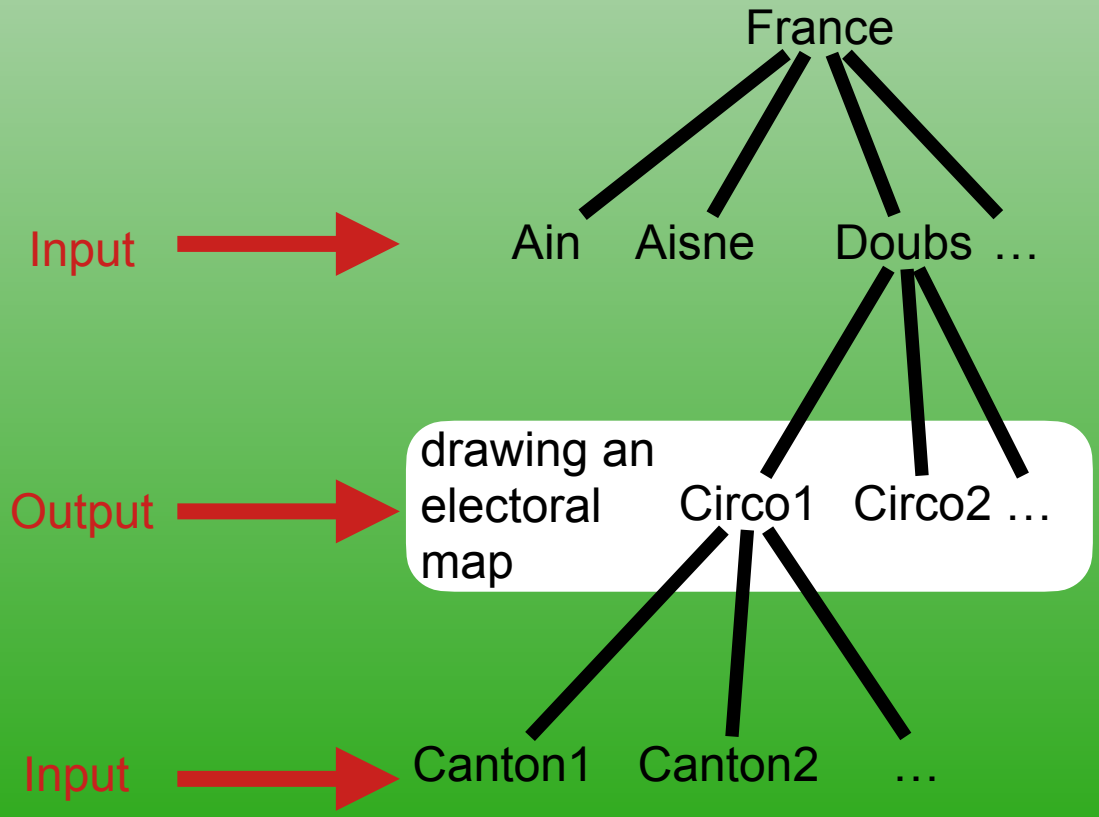
Each département must be partitioned into districts





Each district must be a union of “cantons"





Electoral map(s)

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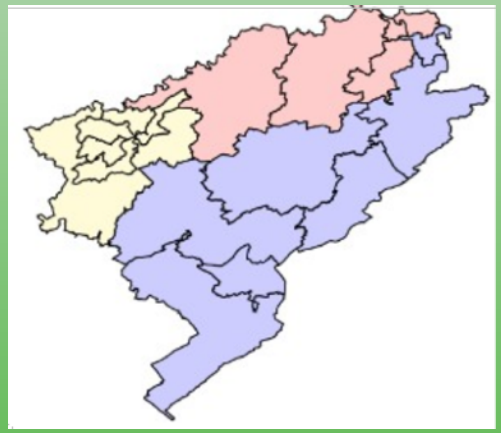
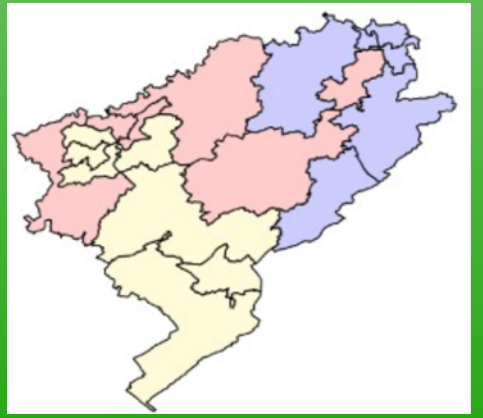


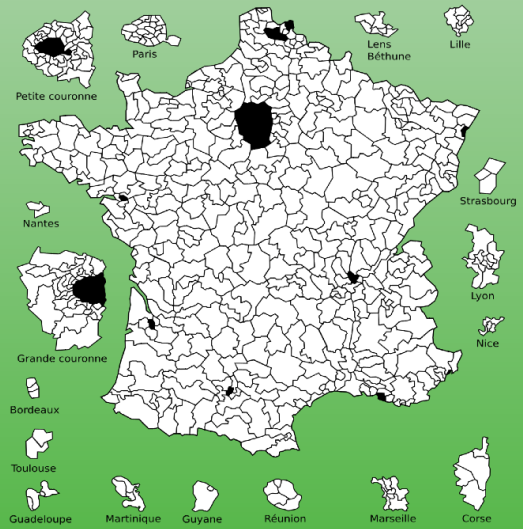
Two possible maps for dép. #25



Demographic balance

- Respect geographical constraints
  - → be included in a departement
  - → be a union of cantons
  - → be connected.

- Respect a balance constraint
  - → In a departement, each district must have same population



Comparing the constraints in France and in the US

- Respect geographical constraints

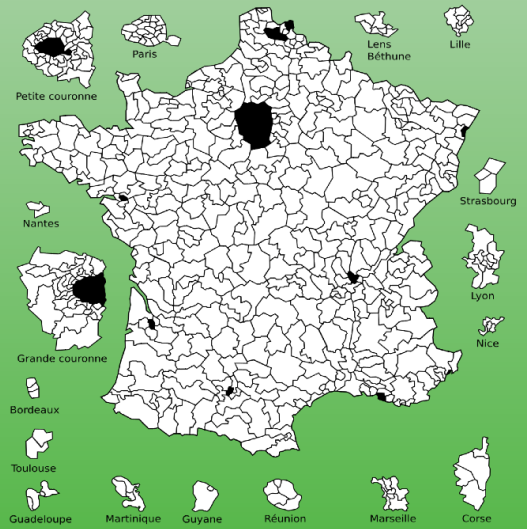
  → be included in a departement (100 in France, 50 in the US)

  → be a union of cantons (30 in each departement, 160 000 in the US)

  → be connected.

- Respect a balance constraint

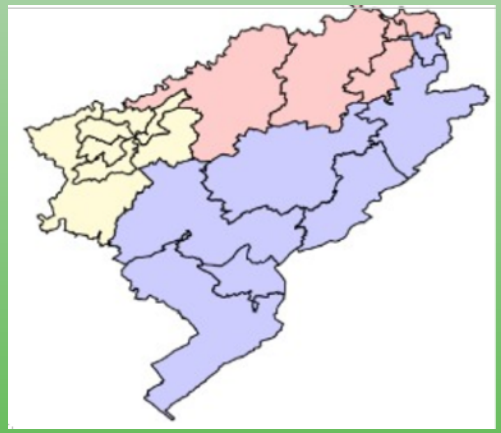
  → In a departement, each district must have same population (up to 20% in France, almost 0% in the US)



Granularity level: in France, combinatorial problem (much more 'discrete' than in USA)

Question

- How do geographical constraints align with the demographic balance constraint/goal ?



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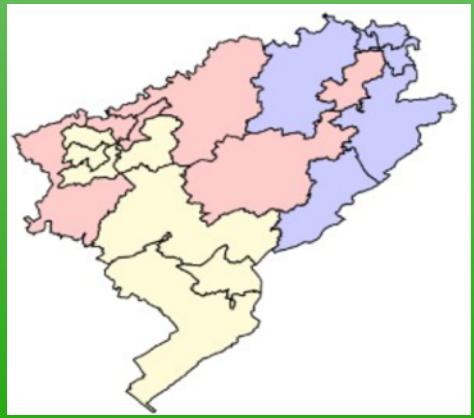


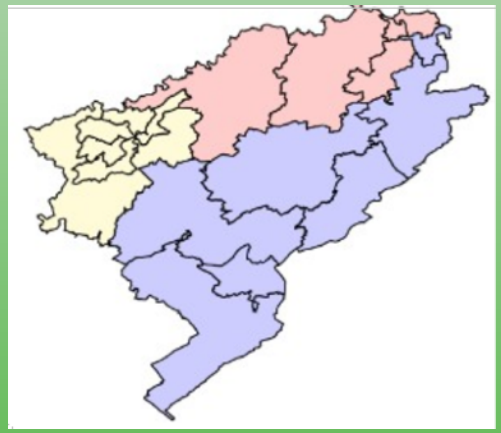
Question

- How do geographical constraints align with the demographic balance constraint/goal ?

- How do they restrict the set of possible maps ?

  → Are there many ?

  → Does this reduce the ability to gerrymander ?



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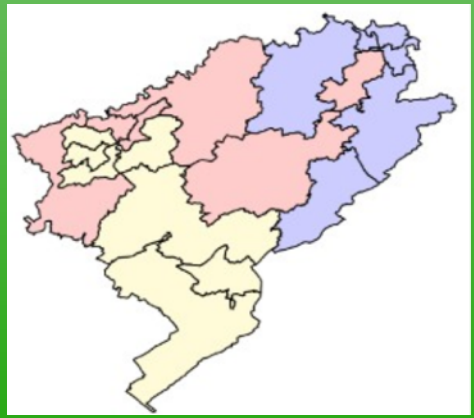


Demographic (un)balance

Demographic balance constraint:

Département i, with population P, and d_i districts → average

$$\tilde{P} = \frac{P}{d_j}$$

The population P_j of each district D_j must satisfy :

$$0.8\tilde{P} \leqslant P_j \leqslant 1.2\tilde{P}$$

- Demographic balance is measured only *inside a département*
- Allowing deviation of 20 %:

  → We can have one district with 80 and one of 120 → 50% more!

  → Why 20 % ?

Demographic (un)balance

Why 20 % ? What is the minimum level of *unbalance* achievable in the département ?

Study on 89 départements (states)

→ For 22 of them : maps with unbalance ≤ 1 % → Almost perfect

→ … But : for 8 of them : NO map with unbalance ≤ 20 %

Min Unbalance (%)	≤ 1	≤ 5	≤ 10	≤ 15	≤ 20	>20
Number of départements	22	44	59	72	81	8

20 % seems to be a good compromise

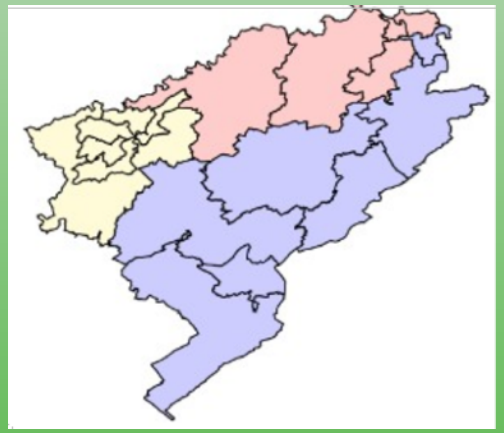
Electoral map(s) : rules

- How do geographical constraints align with the demographic balance constraint/goal ?

- **How do they restrict the set of possible maps ?**

  → **Are there many ?**

  → Does this reduce the ability to gerrymander ?



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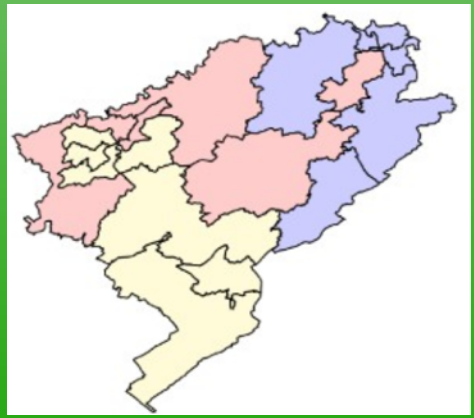


Electoral map(s)

Question : A lot of maps ?

- Expected : exponential, huge number.
- But : **we can actually compute all maps for almost all départements !**

# of maps	0	Between 1 and 100	Between 101 and 1 000	Between 1 001 and 10 000	Between 10 001 and 100 000	More than 100 000
# of départements	8	4	22	29	18	8

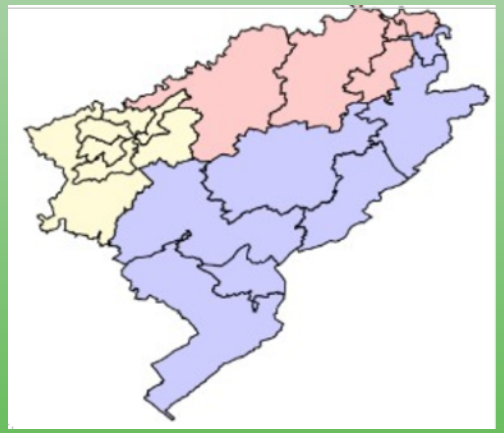
Question

- How do geographical constraints align with the demographic balance constraint/goal ?

- How do they restrict the set of possible maps ?

  → Are there many ? Not really

  → **Does this reduce the ability to gerrymander ?**



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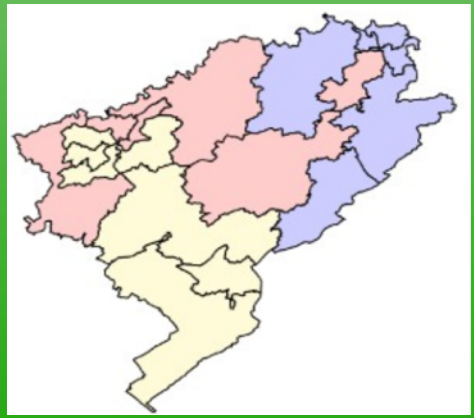
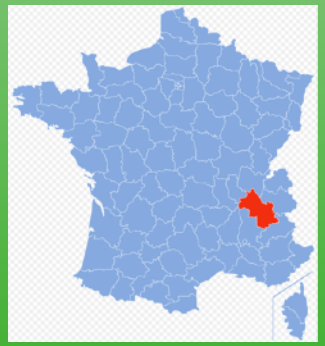


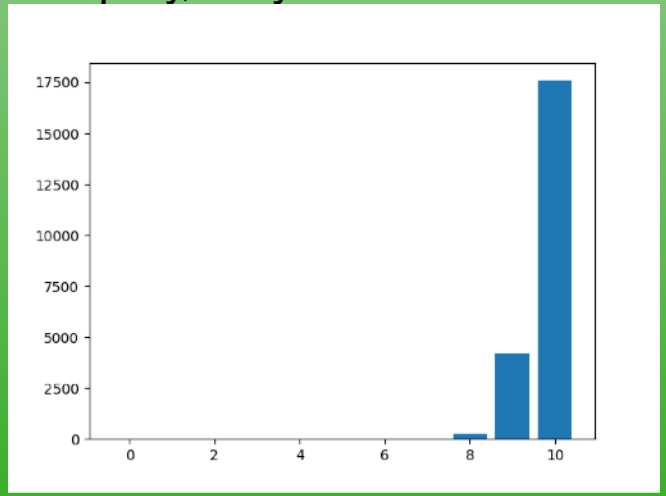
Our approach : diversity of results

- Ideally :

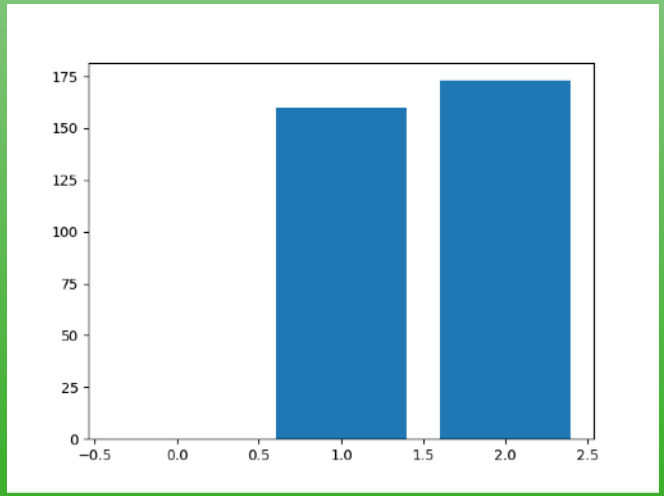
  → for all map, compute the result of elections

  → For each party, study distribution of the results

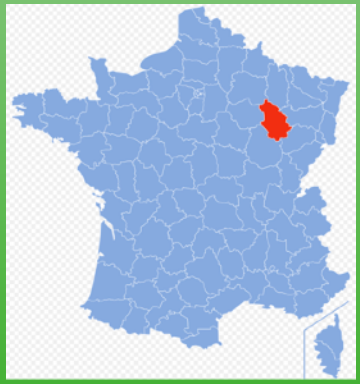




NUPES results in 38 (Isère)



LR results in 52 (Haute Marne)



Step 1 : simulating elections

- What data?

  → Public access to voting data at each election.

  → Different parties, different alliances every elections

- Key issues

  → Virtual maps means virtual candidates

  ⇒ Assumption : voters vote for parties, not for people

  We use results from 2022 elections
  4 alliances : NUPES, Ensemble, LR, RN

  →No access to preferences for the 2nd round

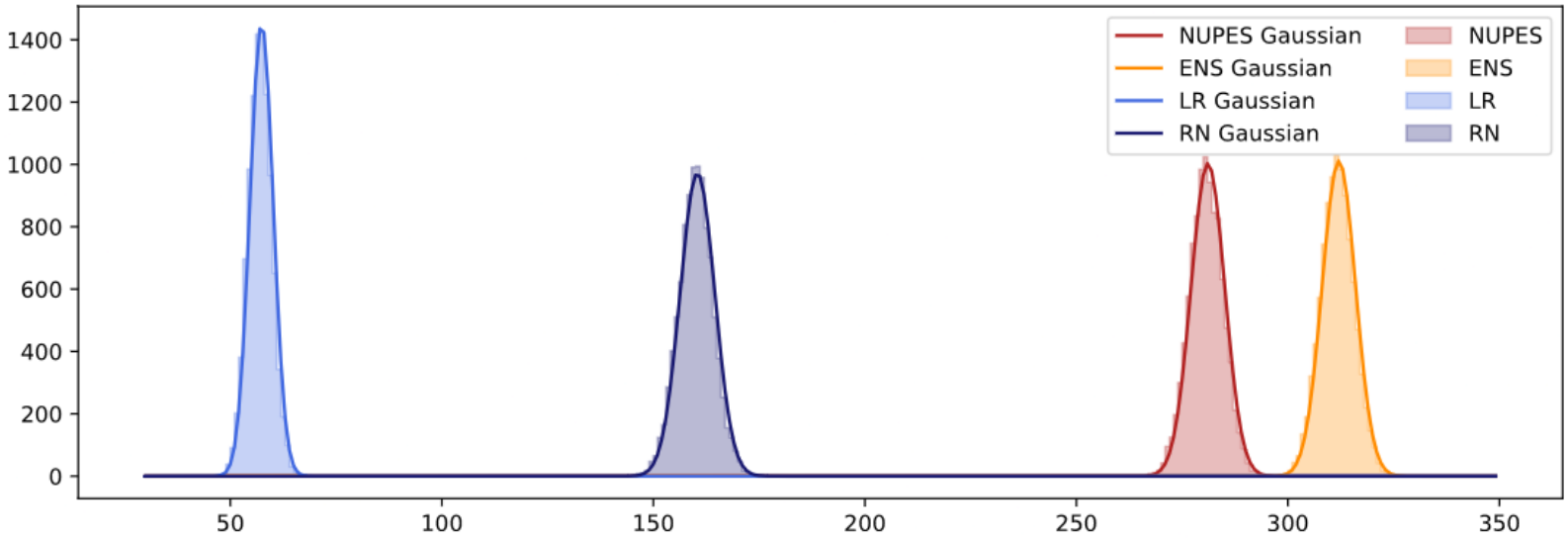
  ⇒ Only consider vote in 1st round, and declare winner any party which would qualify to the 2nd

Step 2a : Distribution of results, national level

- National results : sum of result for each departement
  → Random national result : sum of random variables
  → fast convergence to a gaussian

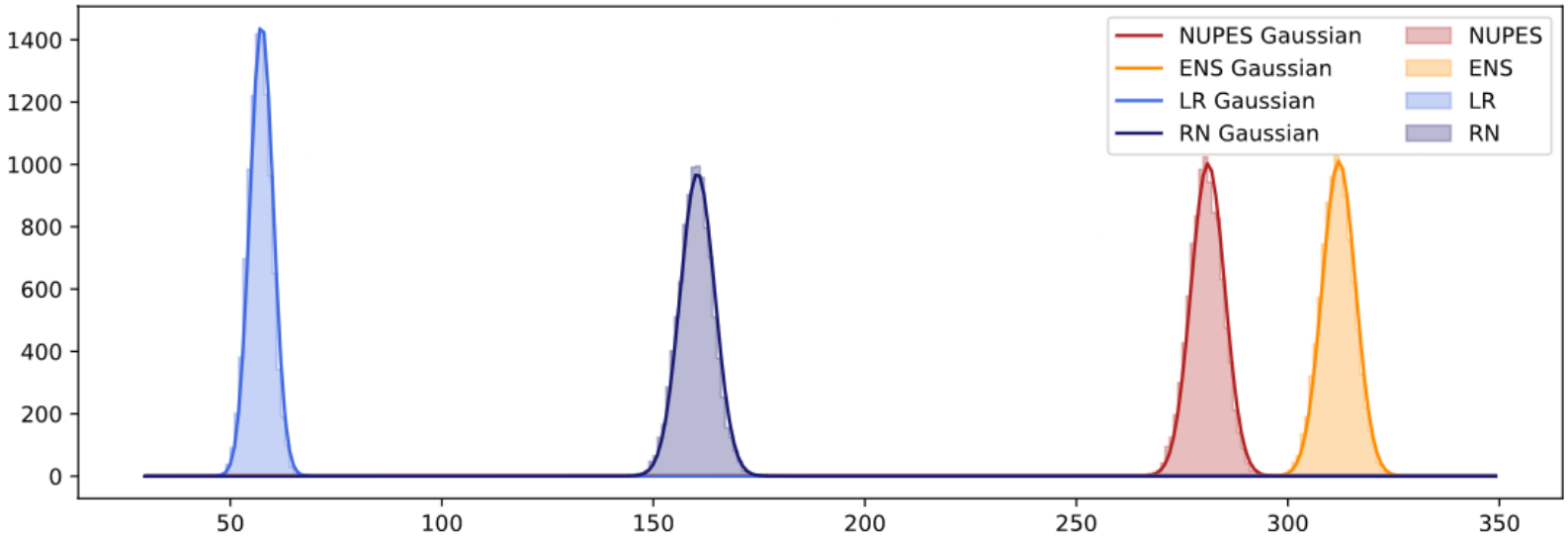
- To estimate the histograms of results : draw 10 000 random uniform results in each departement, and aggregate the results

Step 2a : Distribution of results, nat. level



Party	Min	Average	Max	Std. Dev.
NUPES	207	281	330	4
Ensemble	243	312	360	3,9
LR	31	57	96	2,7
RN	101	160	229	4,1

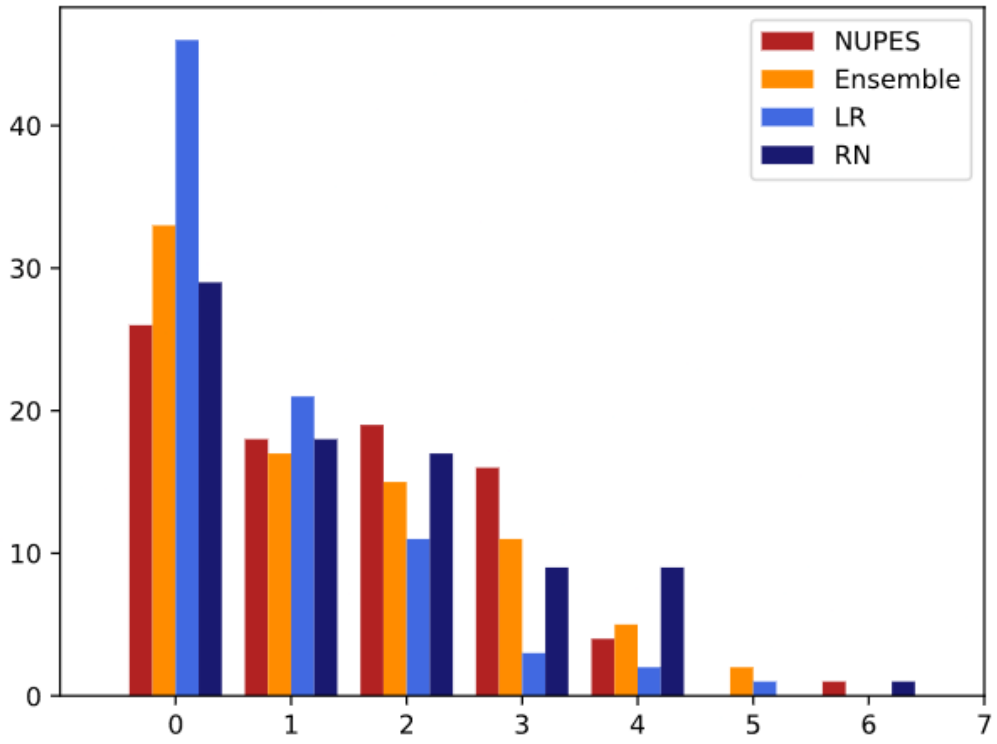
Step 2a : Distribution of results, nat. level



Party	Min	Average	Max	Std. Dev.	2022 results
NUPES	207	281	330	4	258
Ensemble	243	312	360	3,9	299
LR	31	57	96	2,7	63
RN	101	160	229	4,1	178

Looks gaussian, but heavy tails…
And the actual results are in the tails !

Step 2b : Distribution of results, dep. level

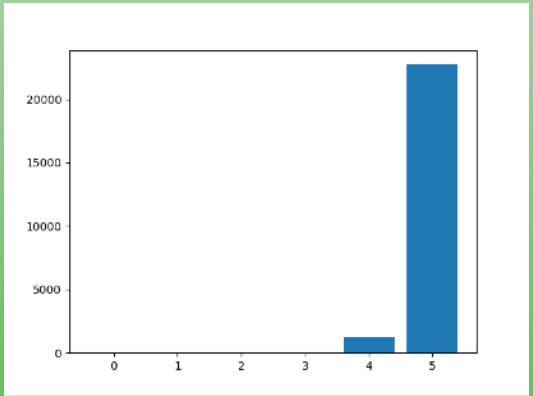


- **Extremal diversity** : difference between best and worst result for a party.
- Pros : simple, easily explainable, captures the magnitude of the potentiality of gerrymandering
- Cons : does not take likeliness into account ; gerrymandering can be more subtle

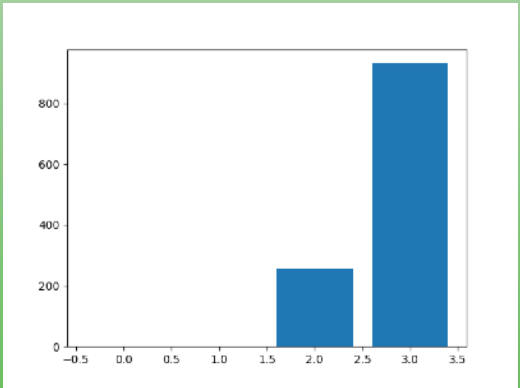
Large diversity of results

x-axis, extremal diversity ; y-axis, number of departement.

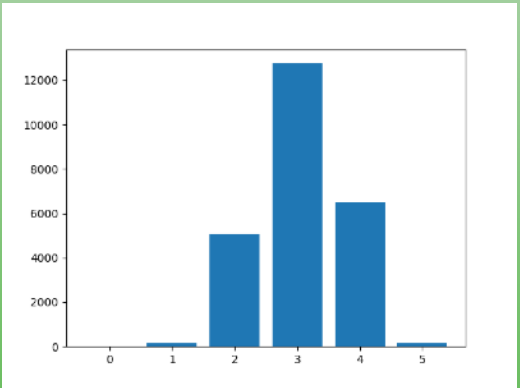
Step 2c : Distribution of results, dep. level



Ensemble results in 37



RN results in 08



NUPES results in 17

- Definition : **gerrymanderable**

  Let be the value of the mode, for one party and one departement. We say the departement is

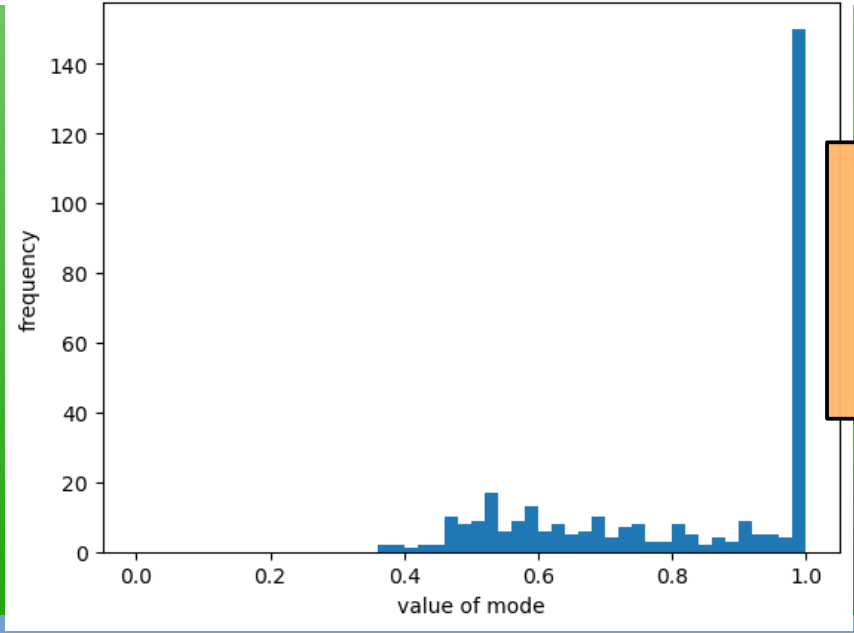
  → *hardly gerrymanderable* if $v > 0,9$

  → *possibly gerrymanderable* if $0,7 < v \leq 0,9$

  → *gerrymanderable* if $v \leqslant 0,7$

Step 2c : Distribution of results, dep. level

	NUPES	ENS	LR	RN
hardly gerrymanderable	38 (45%)	42 (50%)	57 (68%)	36 (43%)
possibly gerrymanderable	14 (17)	10 (12%)	10 (12%)	13 (15%)
gerrymanderable	32 (38%)	32 (38%)	17 (20%)	35 (42%)



In many departements, gerrymandering is obvious
In many others, it is not at all

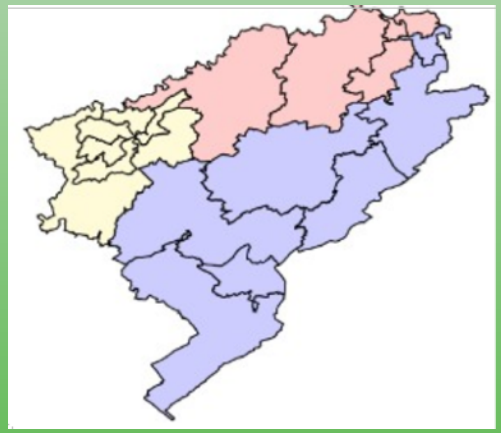
Question

- How do geographical constraints align with the demographic balance constraint/goal ?

- How do they restrict the set of possible maps ?

  → Are there many ? Not really

  → **Does this reduce the ability to gerrymander ?** A bit



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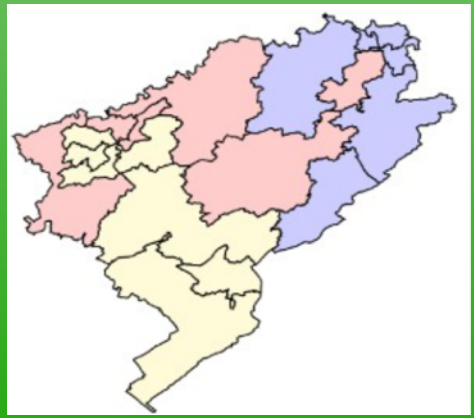

What about the current map ?

- Fix a party p and a map M. We say the map is
  - A ***positive outlier*** for p if the results on M is strictly better than on 90% of the legal maps.
  - a ***negative outlier*** for p if the result on M is strictly worse than on 90% of the legal maps

	NUP	ENS	LR	RN
Positive	4	10	9	10
Negative	13	13	5	1
Gap	- 11 %	- 4 %	5 %	11 %

To conclude

- Redistricting in France : **discrete problem**
- Possible to enumerate all solutions and evaluate the relevance of some rules (e.g. 20%)
- Examine **diversity** of maps to detect the **possibility** of gerrymandering
- Definition of **outliers** based on the distribution of results : **found many**

To conclude

- **Limits of our assumptions :** in practice, voters chose candidates ; and the **second round** may change everything

- CS questions :
  - Use notions from game-theory to detect gerrymandering ?
  - Better use of statistics ?
  - Find *compact* maps ?
  - Sample efficiently ?

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