Linear integer arithmetic and Gaussian elimination

Linear integer arithmetic (Presburger arithmetic) is a logic that allows one to express linear constraints on integers: equalities, inequalities, and divisibility by fixed integers.

In the first half of the talk, we will introduce this logic and three effective approaches for deciding whether a given sentence in it is true. These approaches are rooted in geometry, automata theory, and symbolic computation, respectively.

The second half of the talk will focus on quantifier elimination, the third of these approaches. We will adapt the Gaussian elimination algorithm to systems of inequalities over the integers. We will briefly discuss how this adaptation can be used to show membership in NP of the decision problem of integer programming (a classical result from 1976), as well as of a more challenging problem of integer linear-exponential programming. (Joint work with Alessio Mansutti and Mikhail Starchak, 2024.)