Saket Saurabh, Institute of Mathematical Sciences, Chennai

FPT Approximation Algorithms for Coverage and Satisfiability Problems

The classical Max Coverage problem is defined as follows: Given a universe U of n elements, a family F of subsets of U, and an integer k, select k sets from F to maximize the number of elements covered. A greedy algorithm achieves a $(1 - 1/e)$-approximation in polynomial time,
and this bound is tight assuming P is not equal to NP. However, from the parameterized complexity viewpoint, Max Coverage is W[2]-hard when parameterized by k, and it is known to be hard to approximate within any constant factor in time $f(k).n^{o(k)}$, assuming the Exponential Time Hypothesis (ETH). This rules out the possibility of constant-factor FPT approximation algorithms in general.

To circumvent this hardness, we explore approximation schemes tailored to well-structured instances of the problem. An Efficient Polynomial-Time Approximation Scheme (EPAS) is an algorithm that, for every $\varepsilon > 0$, returns a $(1 + \varepsilon)$-approximate solution in time
$f(k, e).n^{O(1)}$. A Parameterized Approximation Scheme (PAS) also returns a $(1 + \varepsilon)$-approximate solution, but allows the dependence on ε in the exponent of n; that is, its running time is of the form $f(k, e).n^{g(\varepsilon)}$ for some function g.

In this talk, we list recent developments in designing EPASes and PASes for Max Coverage and its variants over well-structured set systems. These include families with bounded VC-dimension, bounded intersection complexity, or geometric and graph-based constraints. We describe algorithmic techniques that exploit such structures to obtain efficient approximation schemes. We also highlight connections to related problems in partial coverage, capacitated coverage, connected variants, and structurally restricted Max-SAT.