The Success Story of Polyregular Functions: Why We Need Pebbles

Regular functions are a robust class of string-to-string functions, one of whose characterisations is that they are exactly the functions recognisable by deterministic two-way transducers. This implies that the mapping describing the output length in terms of the input length (the growth) is always a linear function. To go beyond linear growth, one can equip the transducers with multiple reading heads (pebbles). The number of pebbles constitutes a bound on the degree of the polynomial describing the growth. The functions recognised by these pebble automata are called polyregular.

Over the past years, the properties of polyregular functions have been studied extensively. Multiple equivalent characterisations have been found, and variants of the corresponding models have been investigated. In my talk, I give an introduction to the realm of polyregular functions by discussing some of those characterisations, recent developments, and the parameters in the models that are linked to the growth. With simple constructions, I will explain the asymmetry of the connection between the growth degree and the number of pebbles. That is, in general, the growth degree of a polyregular function does not bound the minimum number of pebbles needed in a transducer to compute the function.

The talk is based on my DLT 2024 survey paper and on work with Lê Thành Dũng (Tito) Nguyễn and Cécilia Pradic, as well as with Mikołaj Bojańczyk and Nathan Lhote.