Generalizing Roberts' characterization of unit interval graphs

For any natural number d, a graph G is a (disjoint) d-interval graph if it is the intersection graph of (disjoint) d-intervals, the union of d (disjoint) intervals on the real line. Two important subclasses of d-interval graphs are unit and balanced d-interval graphs (where every interval has unit length or all the intervals associated to a same vertex have the same length, respectively). A celebrated result by Roberts gives a simple characterization of unit interval graphs being exactly claw-free interval graphs. We study the generalization of this characterization for d-interval graphs. In particular, we how that for any $d \ge 2$, if G is a K1, 2d+1-free interval graph, then G is a unit d-interval graph. However, somehow surprisingly, under the same assumptions, G is not always a disjoint unit d-interval graphs. This implies that the class of disjoint unit d-interval graphs is strictly included in the class of unit d-interval graphs.