Resolving Nondeterminism with Randomness

In automata theory, nondeterminism is a fundamental paradigm that can offer succinctness and expressivity, often at the cost of computational complexity. While determinisation provides a standard route to solving many common problems in automata theory, some weak forms of nondeterminism can be dealt with in some problems without costly determinisation. For example, the handling of specifications given by nondeterministic automata over infinite words for the problems of reactive synthesis or runtime verification requires resolving nondeterministic choices without knowing the future of the input word. We define and study classes of ω -regular automata for which the nondeterminism can be resolved by a policy that uses a combination of memory and randomness on any input word, based solely on the prefix read so far.

We examine two settings for providing the input word to an automaton. In the first setting, called adversarial resolvability, the input word is constructed letter-by-letter by an adversary, dependent on the resolver's previous decisions. In the second setting, called stochastic resolvability, the adversary precommits to an infinite word and reveals it letter-by-letter. In each setting, we require the existence of an almost-sure resolver, i.e., a policy that ensures that as long as the adversary provides a word in the language of the underlying nondeterministic automaton, the run constructed by the policy is accepting with probability 1.

The class of automata that are adversarially resolvable is the well-studied class of history-deterministic automata. The case of stochastically resolvable automata, on the other hand, defines a novel class. Restricting the class of resolvers in both settings to stochastic policies without memory introduces two additional new classes of automata. We show that the new automaton classes offer interesting trade-offs between succinctness, expressivity, and computational complexity, providing a fine gradation between deterministic automata and nondeterministic automata.